Example: Determine the absolute and relation errors, when approximating x by x^* for the following:

1. $x = 0.3000 \times 10^1$ and $x^* = 0.3100 \times 10^1$
2. $x = 0.3000 \times 10^{-3}$ and $x^* = 0.3100 \times 10^{-3}$
3. $x = 0.3000 \times 10^4$ and $x^* = 0.3100 \times 10^4$

Solution:

1. $e_x = |x - x^*| = 0.1 \times 10^0$ $R_x = \frac{e_x}{|x|} = 0.33333 \times 10^{-1}$

2. H.W 3. H.W.

Effect of round – off error on the operation of arithmetic

Let x^* and y^* are approximations of x and y respectively:

1.The addition

$$e_{(x+y)} = |(x+y) - (x^* + y^*)| = |x - x^* + y - y^*|$$

$$e_{(x+y)} \leq |x - x^*| + |y - y^*| = e_x + e_y$$

$$\therefore e_{(x+y)} \leq e_x + e_y$$

$$R_{(x+y)} = \frac{e_{(x+y)}}{|x+y|} \leq \frac{1}{|x+y|}(e_x + e_y) * \frac{|x.y|}{|x.y|}$$

$$R_{(x+y)} = \frac{|x||y|}{|x+y|}\left(\frac{e_x}{|x||y|} + \frac{e_y}{|x||y|}\right)$$

$$R_{(x+y)} = \frac{|x||y|}{|x+y|}\left(\frac{1}{|y|}R_x + \frac{1}{|x|}R_y\right)$$

$$R_{(x+y)} \leq \frac{1}{|x+y|}(|x|R_x + |y|R_y)$$