2. The subtract :

$$e_{(x-y)} \leq e_x - e_y$$

$$R_{(x-y)} \leq \frac{1}{|x+y|}\left(|x|R_x + |y|R_y\right)$$

$$R_{(x-y)} = \frac{e_{(x-y)}}{|x-y|} \leq \frac{1}{|x-y|}\left(e_x - e_y\right) * \frac{|x.y|}{|x.y|}$$

$$R_{(x-y)} = \frac{|x.y|}{|x-y|}\left(\frac{e_x}{|x||y|} - \frac{e_y}{|x||y|}\right)$$

$$R_{(x-y)} = \frac{|x||y|}{|x-y|}\left(\frac{e_x}{|x||y|} - \frac{e_y}{|x||y|}\right)$$

$$R_{(x-y)} = \frac{|x||y|}{|x-y|}\left(\frac{1}{|y|}R_x - \frac{1}{|x|}R_y\right)$$

$$R_{(x-y)} \leq \frac{1}{|x-y|}\left(|x|R_x - |y|R_y\right)$$

Example: find the bound of exact value 1. x+y 2. x-y if

$x^* = 23.86$ and $y^* = 0.01762$ where x and y rounding four-digit. (H.W).

3. Multipliction:

$$e_x = |x - x^*| = \begin{cases} x - x^*; x \geq x^* \\ -(x - x^*); x < x^* \end{cases}$$

$$e_y = |y - y^*| = \begin{cases} y - y^*; y \geq y^* \\ -(y - y^*); y < y^* \end{cases}$$

$e_{(x.y)} = |x.y - x^*.y^*|$

1. If $x \geq x^*$ and $y \geq y^*$

$e_x = x - x^*$ and $e_y = y - y^*$

$x = e_x + x^*$ and $y = e_y + y^*$

$e_{(x.y)} = \left|(e_x + x^*)(e_y + y^*) - x^*.y^*\right|$

$e_{(x.y)} = \left|e_x e_y + x^* e_y + y^* e_x + x^* y^* - x^* y^*\right|$

$e_{(x.y)} = \left|e_x e_y + x^* e_y + y^* e_x\right|$

$e_{(x.y)} \leq e_x e_y + |x^*| e_y + |y^*| e_x$

Since $0 < e_x < 1$ and $0 < e_y < 1 \Rightarrow e_x e_y \cong 0$

$$\therefore e_{(x.y)} = |x^*| e_y + |y^*| e_x$$