4. The Division

Let $e_x = |x - x^*| = \begin{cases} x - x^* & ; x \geq x^* \\ -(x - x^*) & ; x < x^* \end{cases}$

$$e_y = |y - y^*| = \begin{cases} y - y^* & ; y \geq y^* \\ -(y - y^*) & ; y < y^* \end{cases}$$

$$e_{x/y} = \left|\frac{x}{y} - \frac{x^*}{y^*}\right|$$

1. If $x \geq x^*$ and $y \geq y^*$

$e_x = x - x^*$ and $e_y = y - y^*$

$x = e_x + x^*$ and $y = e_y + y^*$

$$e_{x/y} = \left|\frac{e_x + x^*}{e_y + y^*} - \frac{x^*}{y^*}\right| = \left|\frac{e_x + x^*}{y^*\left(\frac{e_y}{y^*} + 1\right)} - \frac{x^*}{y^*}\right|$$

$$e_{x/y} = \left|\left(\frac{e_x}{y^*} + \frac{x^*}{y^*}\right)\left(1 + \frac{e_y}{y^*}\right)^{-1} - \frac{x^*}{y^*}\right| \ldots \quad (1)$$

$$\left(1 + \frac{e_y}{y^*}\right)^{-1} = 1 - \frac{e_y}{y^*} + \frac{e_y^2}{y^*} - \frac{e_y^3}{y^*} + \cdots \qquad (2) \quad (binomul\ theorem)$$

Since $0 < e_y < 1 \;\; \Rightarrow e_y^2 \cong 0\,, e_y^3 \cong 0\,, \ldots$

Substitute (2) in to (1) we have

$$e_{x/y} = \left|\left(\frac{e_x}{y^*} + \frac{x^*}{y^*}\right)\left(1 + \frac{e_y}{y^*}\right) - \frac{x^*}{y^*}\right|$$

$$e_{x/y} = \left|\frac{e_x}{y^*} - \frac{e_x\, e_y}{{y^*}^2} + \frac{x^*}{y^*} - \frac{x^*}{{y^*}^2} e_y - \frac{x^*}{y^*}\right|$$

$$e_{x/y} \leq \frac{e_x}{|y^*|} + \frac{|x^*|}{|{y^*}^2|} e_y$$

$$R_{(x/y)} = \frac{e_{x/y}}{\frac{|x^*|}{|y^*|}} \leq \frac{\frac{e_x}{|y^*|} + \frac{|x^*|}{|y^{*2}|} e_y}{\frac{|x^*|}{|y^*|}}$$

$$R_{(x/y)} \leq R_x + R_y$$