Fuzzy Topological Spaces

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The Lectures

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Contents

Chapter 1

Preliminaries and Introduction

1.1. Fuzzy Set

Zadeh [11], 1965, introduced the concept of fuzzy sets by defining them in terms of mappings from a set into the unit interval on the real line. Fuzzy sets were introduced to provide means to describe situations mathematically which give rise to ill-defined classes, i.e., collections of objects for which there is no precise criteria for membership. Collections of this type have vague or "fuzzy" boundaries; there are objects for which it is impossible to determine whether or not they belong to the collection. The classical mathematical theories, by which certain types of certainty can be expressed, are the classical set theory and the probability theory. In terms of set theory, uncertainty is expressed by any given set of possible alternatives in situations where only one of the alternatives may actually happen. Uncertainty expressed in terms of sets of alternatives results from the nonspecificity inherent in each set. Probability theory expresses uncertainty in terms of a classical measure on subsets of a given set of alternatives. The set theory, introduced by Zadeh, presents the notion that membership in a given subset is a matter of degree rather than that of totally in or totally out. With fuzzy set theory, one obtains a logic in which statements may be true or false to different degrees rather than the bivalent situation of being true or false; consequently, certain laws of bivalent logic do not hold, e.g. the law of the excluded middle and the law of contradiction. This results in an enriched scientific methodology. Chang

[2], introduced the notion of a fuzzy topology of a set in 1968, and our work is based on the study of the properties of fuzzy topological spaces.

Definition 1.1.1 [11]. Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples $A = \{(x, \mu_A(x)) : x \in X\}$.

1.2. Basic Operations on Fuzzy Sets

Definition 1.2.1 [11]: Let $A = \{(x, \mu_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x)) : x \in X\}$ be two fuzzy sets in X. Then their union $A \vee B$, intersection $A \wedge B$ and complement A^c are also fuzzy sets with the membership functions defined as follows:

(i) $\mu_{A\vee B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \ \forall x \in X.$

(ii) $\mu_{A\wedge B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \ \forall x \in X.$

(iii) $\mu_{A^c}(x) = 1 - \mu_A(x), \ \forall x \in X.$

Further,

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x), \ \forall x \in X.$

(b) $A = B$ iff $\mu_A(x) = \mu_B(x), \ \forall x \in X.$

Lemma 1.2.2 [11]: The De Morgan's law are true for fuzzy sets. That is suppose $A = \{(x, \mu_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x)) : x \in X\}$ are fuzzy sets, then

$$(A \cup B)^c = A^c \cap B^c \cdots\cdots\cdots\cdots (1)$$

$$(A \cap B)^c = A^c \cup B^c \cdots\cdots\cdots\cdots (2)$$