Proposition 1.3.8 [6]. Let $f_j : X_j \to Y_j$ be mappings and A_j be fuzzy sets of Y_j, $j = 1, 2$; then $(f_1 \times f_2)^{-1}(A_1 \times A_2) = f_1^{-1}(A_1) \times f_2^{-1}(A_2)$.

Proof. For each $(x_1, x_2) \in X_1 \times X_2$, we have $(f_1 \times f_2)^{-1}(A_1 \times A_2) = (A_1 \times A_2)(f_1(x_1), f_2(x_2)) = min(A_1 f_1(x_1), A_2 f_2(x_2)) = min(f_1^{-1}(A_1)(x_1), f_2^{-1}(A_2)(x_2)) = (f_1^{-1}(A_1) \times f_2^{-1}(A_2))(x_1, x_2)$.

Proposition 1.3.9 [6]. Let $g : X \to X \times Y$ be the graph of a mapping $f : X \to Y$. Let A be a fuzzy set of X and B be a fuzzy set of Y, then $g^{-1}(A \times B) = A \wedge f^{-1}(B)$.

Proof. For each $x \in X$, we have $g^{-1}(A \times B)(x) = (A \times B)g(x) = (A \times B)(x, f(x)) = min(A(x), B(f(x))) = (A \wedge f^{-1}(B))(x)$.

Chapter 2

Fuzzy Topological Space

2.1. Fuzzy Topological Space

Definition 2.1.1 [6]. A family $\tau \subseteq I^X$ of fuzzy sets is called a fuzzy topology for X if it satisfies the following three axioms:

(1) $\overline{0}, \overline{1} \in \tau$.

(2) $\forall A, B \in \tau \Rightarrow A \wedge B \in \tau$.

(3) $\forall (A_j)_{j \in J} \in \tau \Rightarrow \vee_{j \in J} A_j \in \tau$.

The pair (X, τ) is called a fuzzy topological space or fts, for short. The elements of τ are called fuzzy open sets. A fuzzy set K is called fuzzy closed if $K^c \in \tau$. We denote by τ^c the collection of all fuzzy closed sets in this fuzzy topological space. Obviously, we have:

(a) $\alpha^c \in \tau^c$,

(b) if $K, M \in \tau^c$, then $K \vee M \in \tau^c$ and

(c) if $\{K_j : j \in J\} \in \tau^c$, then $\wedge\{K_j : j \in J\} \in \tau^c$.

Example 2.1.2 [6]. Let $X = \{a, b\}$. Let A be a fuzzy set on X defined as $A(a) = 0.5, A(b) = 0.4$. The $\tau = \{\overline{0}, A, \overline{1}\}$ is a fuzzy topology. (X, τ) is a fuzzy topological space. $\overline{0}(a) = 0, \forall a \in x, \overline{1}(a) = 1, \forall a \in x$.

Let τ_1 and τ_2 be two fuzzy topology for X. If the inclusion relation $\tau_1 \subset \tau_2$ holds, we say that τ_2 is finer than τ_1 and τ_1 is coarser than τ_2.

2.2 Base and Subbase for FTS

Definition 2.2.1 [1]. A base for a fuzzy topological space (X,τ) is a sub collection B of τ such that each member A of τ can be written as $A = \vee_{j\in\Lambda} A_j$, where each $A_j \in B$.

Definition 2.2.2 [1]. A subbase for a fuzzy topological space(X,τ) is a subcollection S of τ such that the collection of infimum of finite subfamilies of S forms a base for (X,τ).

Definition 2.2.3. Let (X,τ) be an fts. Suppose A is any subset of X. Then (A,τ_A) is called a fuzzy subspace of (X,τ), where $\tau_A = \{B_A : B \in \tau\}$, $B = \{(x,\mu_B(x)) : x \in X\}$ and $B_A = \{(x,\mu_{B|A}(x)) : x \in A\}$.

Definition 2.2.4 [6]. A fuzzy point P in X is a special fuzzy set with membership function defined by

$$P(x) = \begin{cases} \lambda & \text{if } x = y, \\ 0 & \text{if } x \neq y; \end{cases}$$

where $0 < \lambda \leq 1$. P is said to have support y, value λ and is denoted by P_y^λ or $P(y,\lambda)$.

Let A be a fuzzy set in X, then $P_y^\alpha \subset A \Leftrightarrow \alpha \leq A(y)$. In particular, $P_y^\alpha \subset P_z^\beta \Leftrightarrow y = z, \alpha \leq \beta$. A fuzzy point P_y^α is said to be in A, denoted by $P_y^\alpha \in A \Leftrightarrow \alpha \leq A(y)$.

The complement of the fuzzy point P_x^λ is denoted either by $P_x^{1-\lambda}$ or by $(P_x^\lambda)^c$.

Definition 2.2.5 [6]. The fuzzy point P_x^λ is said to be contained in a fuzzy set A, or to belong to A, denoted by $P_x^\lambda \in A$ if and only if $\lambda < A(x)$.

Every fuzzy set A can be expressed as the union of all the fuzzy points which belong to A. That is, if $A(x)$ is not zero for $x \in X$, then $A(x) = sup\{\lambda : P_x^\lambda, 0 < \lambda \leq A(x)\}$.

Definition 2.2.6 [6]. Two fuzzy sets A, B in X are said to be intersecting if and only if there exists a point $x \in X$ such that $(A \wedge B)(x) \neq 0$. For such a case, we say that A and B intersect at x.