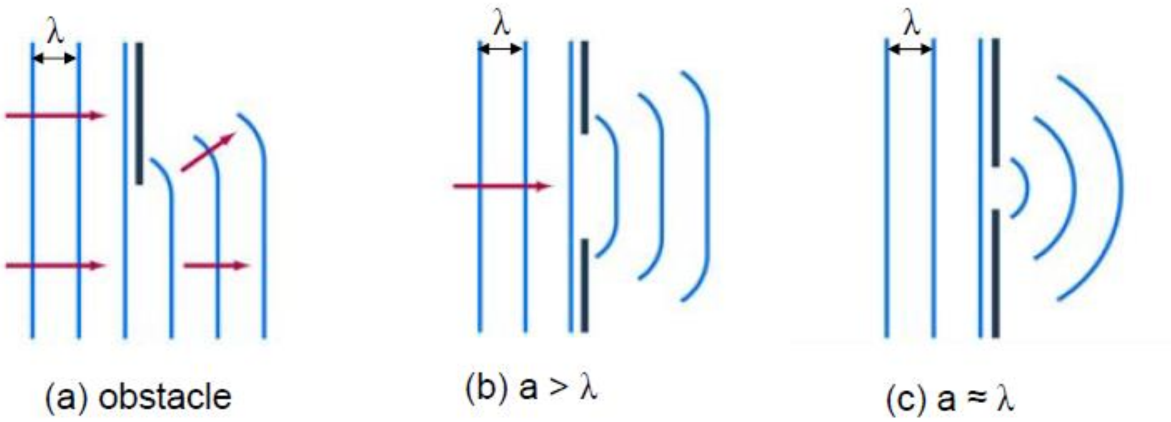
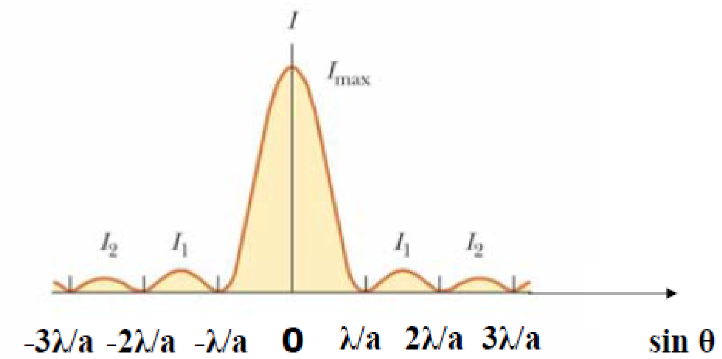
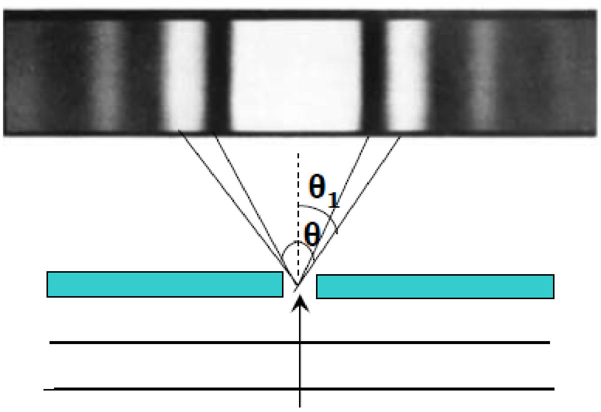
1.5.Diffraction of Light Wave

- Definition – is defined as *the bending of waves as they travel around obstacles or pass through an aperture comparable to the wavelength of the waves.*
- For examples:

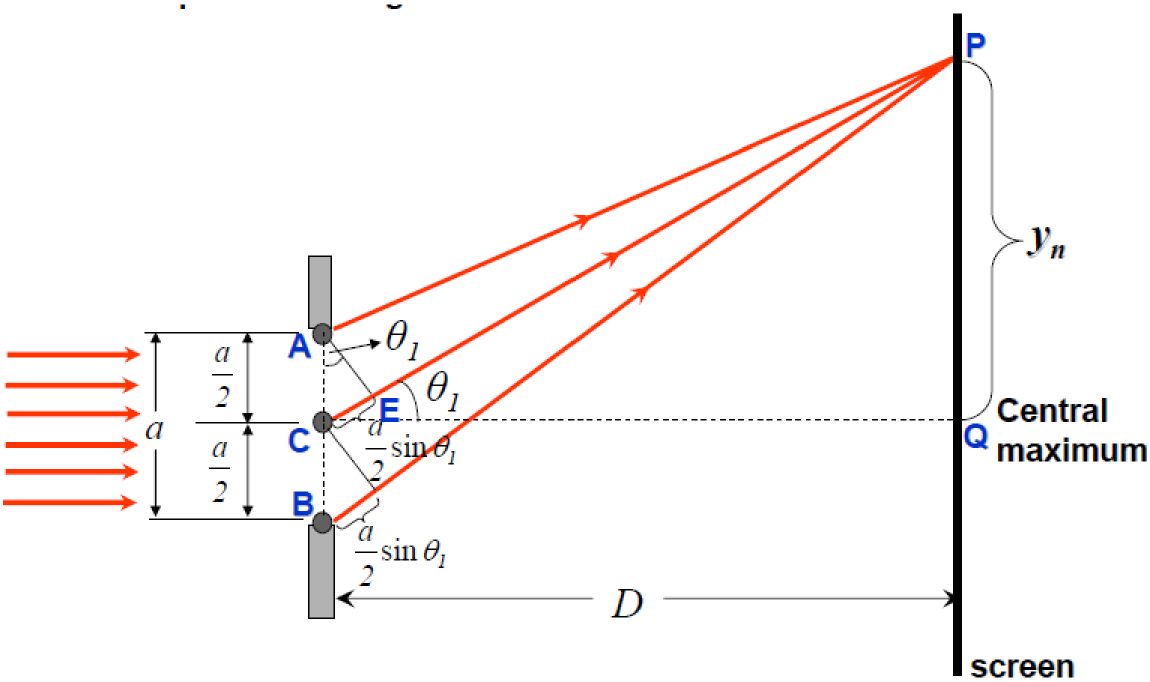


2.5.Diffraction by a Single Slit





- The central fringe is a bright fringe (central maximum).
- Other rays with angle θ and θ_1 will produce minimum and maximum on both sides of the central maximum.



- The slit is split into two equal parts, AC and CB. A,C and B are new sources of secondary wavelets. (Huygen's principle)
- When the wavefronts from A, C and B superpose, interference will occur at P.

- As AB is very small, thus AE is perpendicular to CP and AP = EP, and therefore the path difference at P between ray AP and CP is :

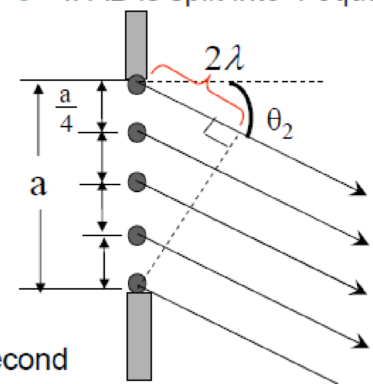
$$\text{path difference} = CE = \frac{a}{2}\sin\theta_1$$

- If the first minimum (first order) is at P, hence :

$$\text{path difference} = \frac{a}{2}\sin\theta_1 = \frac{\lambda}{2}$$

$$a\sin\theta_1 = \lambda$$

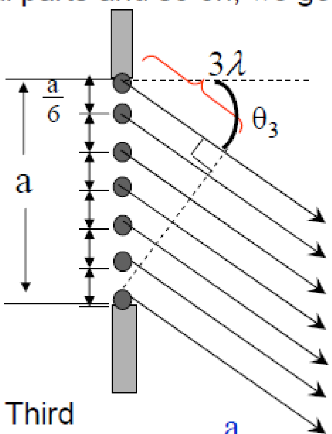
- If AB is split into 4 equal parts, 6 equal parts and so on, we get:



Second minimum (2nd order minimum)

$$\frac{a}{4}\sin\theta_2 = \frac{\lambda}{2}$$

$$a\sin\theta_2 = 2\lambda$$



Third minimum (3rd order minimum)

$$\frac{a}{6}\sin\theta_3 = \frac{\lambda}{2}$$

$$a\sin\theta_3 = 3\lambda$$

Example 1:

A monochromatic light of wavelength 6 x10^{-7} m passes through a single slit of width 2 x 10^{-6} m.

a. Calculate the width of central maximum:

i. in degrees;

ii. in centimetres, on a screen 5 cm away from the slit

b. Find the number of minimum that can be observed.

Solution: $\lambda = 6 \times 10^{-7}\ m,\ a = 2 \times 10^{-6}\ m$

a. i. $a \sin\theta_n = n\lambda; \quad n = 1$

$$\theta_1 = 17.46^\circ$$

$$\text{The width of central max.,} 2\theta_1 = 2x17.46^\circ = 34.96^\circ$$

ii. Given $D = 5x10^{-2}\ m$

$$y_n = \frac{n\lambda D}{a}; \quad n = 1$$

$$y_1 = \frac{\lambda D}{a} = 0.015\ m$$

7 $\text{The width of central max.,} 2y_1 = 2x0.015 = 0.030\ m = 3.0\ cm$

b. $a \sin\theta_n = n\lambda$

$$\text{For maximum no. of } n, \theta = 90^\circ$$

$$a \sin 90^\circ = n\lambda$$

$$n = \frac{a}{\lambda} = 3.33$$

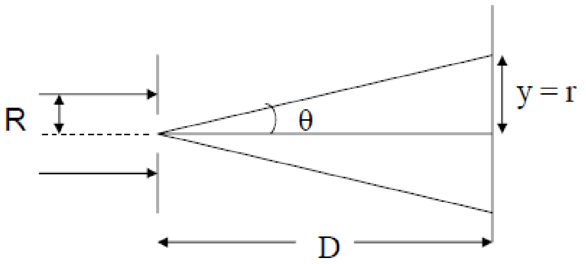
$$\text{maximum order, } n = 3$$

Thus the number of minimum that can be observed is 6.

Example 2:

A beam of a monochromatic light of wavelength 600 nm passes through a single slit of width 3 x 10^{-3} mm. The beam of light has a radius of 1.5 mm. Calculate the distance of the screen from the slit so that the radius of the central maximum is 2 times the radius of the light beam.

Solution:



$R = 1.5 \times 10^{-3}\ m$

$a = 3 \times 10^{-3} \times 10^{-3}\ m$

$= 3 \times 10^{-6}\ m$

$\lambda = 600 \times 10^{-9}\ m$

$y_1 = r = 2R = 2(1.5 \times 10^{-3})$

7 $y_1 = 3.0 \times 10^{-3}\ m$ 55

For central maximum :

$$y_n = \frac{n\lambda D}{a}$$

$$\text{when } n = 1, y_1 = \frac{\lambda D}{a}$$

$$D = \frac{y_1 a}{\lambda} = \frac{(3.0\text{x}10^{-3})(3\text{x}10^{-6})}{600\text{x}10^{-9}}$$

$$D = 1.5\text{x}10^{-2}\,\text{m} \ @\ 1.5\,\text{cm}$$

Example 3:

Monochromatic light of wavelength 689 nm falls on a slit. If the angle between first bright fringes on either side of the central maximum is 38°, find the slit width. (Gc.913.4)

Ans. : 3.2 μm

Example 4:

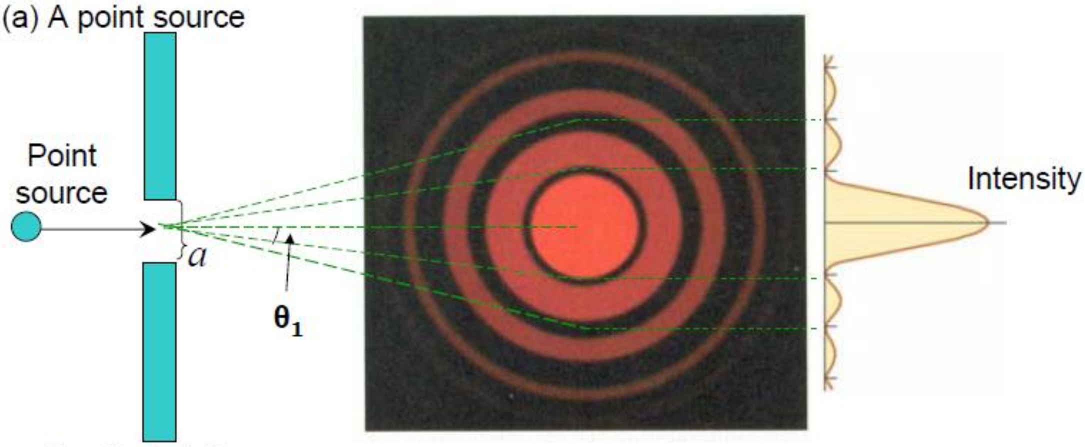
Light of wavelength 633 nm from a distant source is incident on a single slit 0.750 mm wide, and the resulting diffraction pattern is observed on a screen 3.50 m away. Determine the distance between the two dark fringes on either side of the central bright fringe. (Young & freedman,pg.1396.36.4)

Ans. : 5.91 mm 56

3.5.Diffraction by Circular Aperture

- Circular aperture is a circular opening such as a circular lens which light can pass.
- The figures below show the diffraction by the circular aperture for one point source and two point sources.

(a) A point source



For first minimum,

$$\sin\theta_1 = \frac{1.22\lambda}{a}, \quad \text{where } a : \text{diameter of circular aperture}$$

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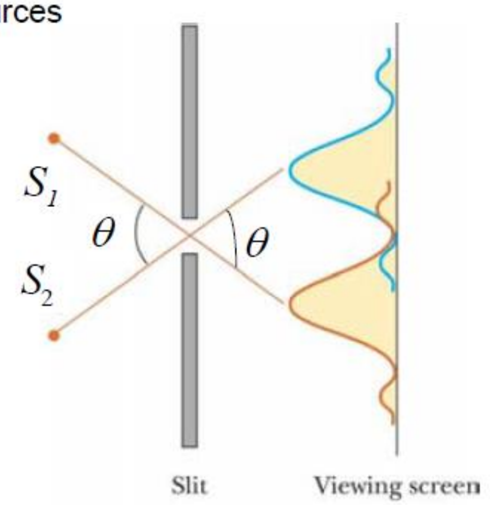
$$\theta_1 : \text{diffraction angle of first minimum ring}$$ 57

(b) Two point sources



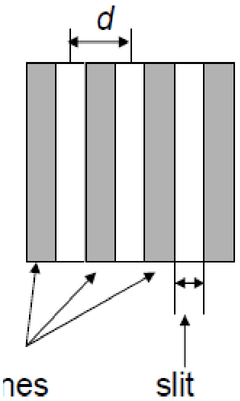
where θ : the angle subtended by two point sources

for example :

7  58

4.5.Diffraction Grating

- Definition – is defined as *a large number of equally spaced parallel slits.*
- Diffraction grating can be made by ruling very fine parallel lines on glass (**transmission grating**) or metal (**reflection grating**) by a very precise machine.
- The untouched spaces between the lines serve as the slits as shown in figure below.



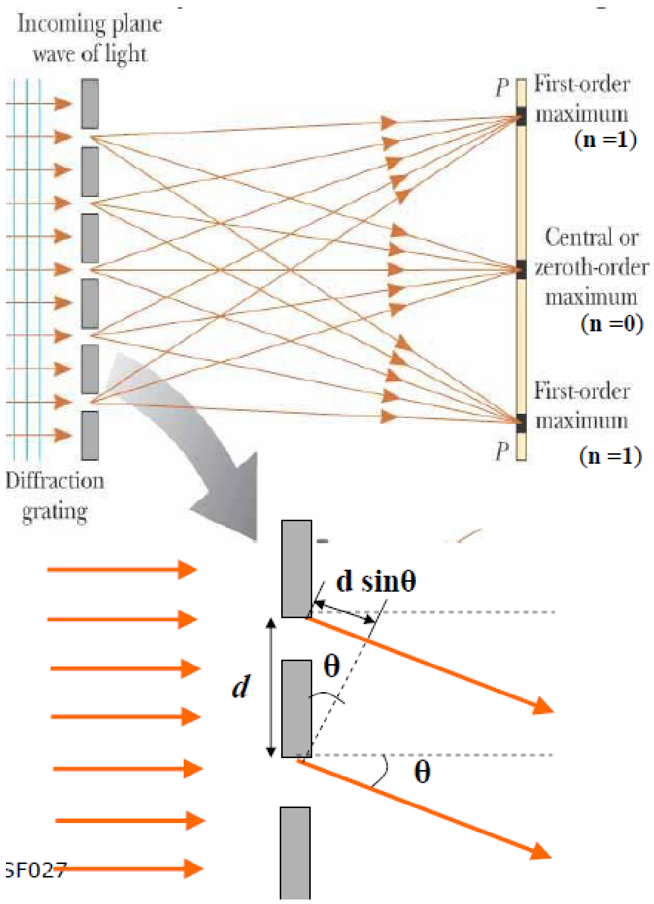
- Light passes through the slit because it is transparent.
- The spaces between the lines are the slits, for example : if there are four lines then we have 3 slits.
- If there N lines per unit length, then slit separation, d is given by:

$$d = \frac{1}{N}$$

e.g. If a diffraction grating has 2000 lines per cm, then

$$d = \frac{1\,cm}{2000} = 5x10^{-4}\,cm \;\; or \; 5x10^{-6}\,m$$

- The light that passes through the slits are coherent .
- Interference pattern is narrower and sharper than double-slits.



- For **constructive Interference (bright):**

Path difference = $n\lambda$

$$d\sin\theta_n = n\lambda,\ n = 0, \pm1, \pm2, \pm3, ..$$

where n : the n[th] order

θ_n: the n[th] angle of diffraction

then $\sin\theta_n = \frac{n\lambda}{d}$

- If $\theta_n = 90^o$, $\sin 90^\circ = \frac{n\lambda}{d}$

$$n_{max} = \frac{d}{\lambda}$$

where n_{max} : maximum number of orders that can be observed.

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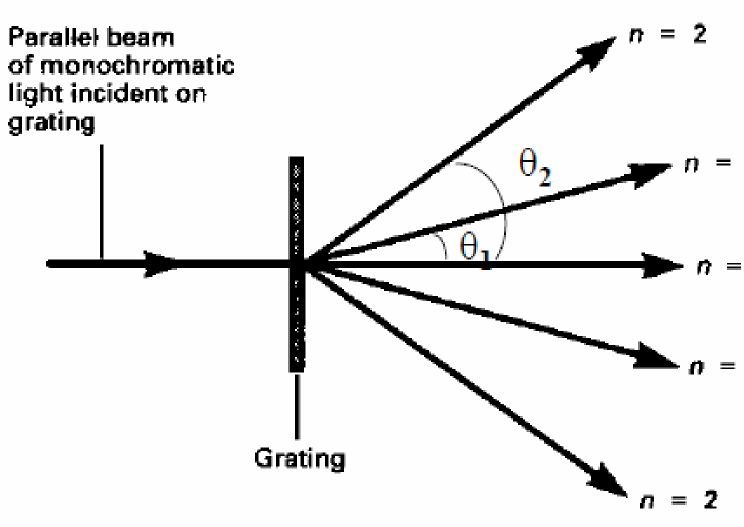
- Figure below shows the diffraction grating pattern.



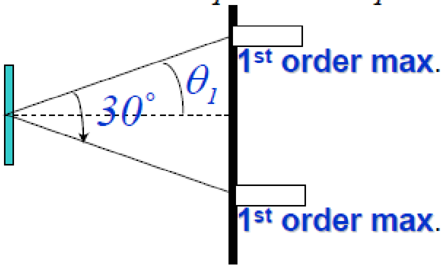
Example 5:

A monochromatic light of wavelength 600 nm incident normally on a diffraction grating. The angle subtended by the first-order maximum lines is 30°.

a. Calculate the number of lines per cm of the grating.

b. What is the angle between the first-order and second-order maximum lines?

<u>Solution</u>: $\lambda = 600 \times 10^{-9}\ m,\ 2\theta_1 = 30^o,\ \theta_1 = 15^o,\ n=1$



a. By applying the equation of diffraction grating for maximum,

$$d \sin\theta_n = n\lambda \quad \text{and} \quad d = \frac{1}{N}$$

$$N = \frac{\sin\theta_1}{(1)\lambda} \Longrightarrow N = 4.31x10^5 \text{ lines per m}$$

or $4.31x10^3$ lines per cm

b. By applying the equation of diffraction grating for maximum,

$$d \sin\theta_n = n\lambda \quad \text{and} \quad d = \frac{1}{N}$$

$$\sin\theta_n = nN\lambda$$

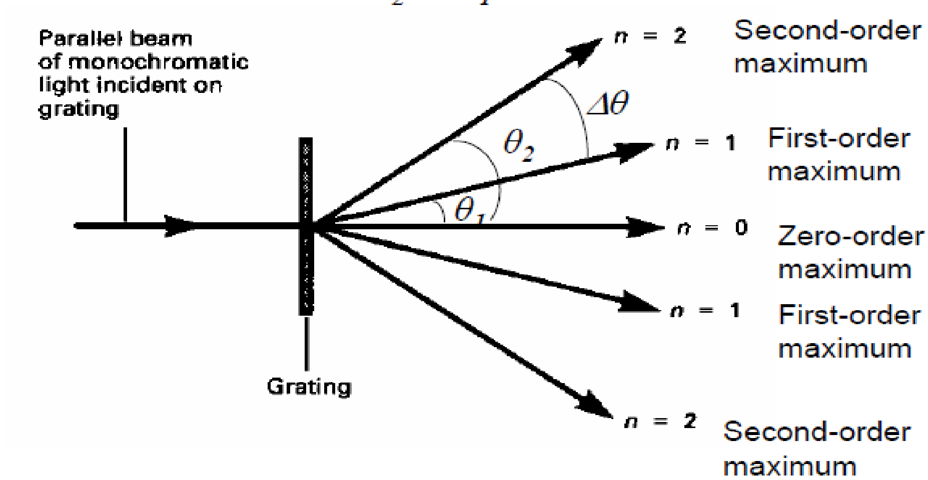
For 2nd order, $n = 2$:

$$\sin\theta_2 = (2)(4.31x10^5)(600x10^{-9})$$

$$\theta_2 = 31.1^\circ$$

thus, the angle between 1st and 2nd order maximum is given by

$$\Delta\theta = \theta_2 - \theta_1 = 16.1^\circ$$



Example 6:

A diffraction grating with 600 lines per mm is illuminated normally with a monochromatic light of wavelength 589 nm. Calculate

a. the angles of the first-order and second-order maximum lines from the zero-order maximum line.

b. the number of orders that can be observed.

Solution: $\lambda = 589 \times 10^{-9}\ m,\ N = 600\ lines\ per\ mm$

a. By applying the equation of diffraction grating for maximum,

$$d \sin\theta_n = n\lambda$$

$$\theta_n = \sin^{-1}\left(\frac{n\lambda}{d}\right)$$

$$\theta_n = \sin^{-1} Nn\lambda$$

For 1st order, $n = 1$:

$$\theta_1 = \sin^{-1}\frac{600\,(1)\,(589\text{x}10^{-9})}{(1\text{x}10^{-3})}$$

$$\theta_1 = 20.7^{\circ}$$

For 1st order, $n = 2$:

$$\theta_2 = \sin^{-1}\frac{600\,(2)\,(589\text{x}10^{-9})}{(1\text{x}10^{-3})}$$

$$\theta_2 = 45^{\circ}$$

b. By applying the equation of diffraction grating for maximum,

$$\sin\theta_n = \frac{n\lambda}{d}$$

For the number of orders can be observed, $\theta = 90^{\circ}$

$$\sin 90^{\circ} = \frac{n\lambda}{d}$$

$$n = \frac{d}{\lambda} = \frac{1}{N\lambda}$$

$$n = \frac{(1\text{x}10^{-3})}{600(589\text{x}10^{-9})}$$

$$n = 2.8$$

$$n = 2$$

hence, the number of orders can be observed is 2.

Example 7: H.W

The first-order maximum line of 589 nm light falling on a diffraction grating is observed at an angle of 15.5°. Determine

a. the slit separation on the grating.

b. the angle of diffraction for third-order maximum line. (Gc.914.32)

Ans. : 2.20 μm, 53.4°

Example 8: H.W

A diffraction grating has 6000 lines per cm. Calculate the angular separation between wavelengths 5.896×10^{-7} m and 5.461×10^{-7} m respectively after transmission through it at normal incidence, in the first-order spectrum (maximum line).

Ans. :1.60°