- Therefore, the separation between central bright fringe with m[th] and (m+1)[th] bright fringe is given by

For the **m[th] bright fringe** :

$$\boxed{y_m = \frac{m\lambda D}{d}}$$

For the **(m+1)[th] bright fringe** :

$$\boxed{y_{m+1} = \frac{(m+1)\lambda D}{d}}$$

- The **separation between successive (consecutive) bright or dark fringes, Δy** is given by

$$\Delta y = y_{m+1} - y_m = \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d}$$

$$\boxed{\Delta y = \frac{\lambda D}{d}}$$

where

m : order $= 0,1,2,.....$

λ : wavelength

D : distance between double - slits and the screen

)27 d : separation between double - slits 24

- The separation between **m[th] dark fringe and central bright fringe** is given by

$$\boxed{x_m = (m + \tfrac{1}{2})\frac{\lambda D}{d}}$$

where

m : order $= 0,1,2,.....$

- From the equation below,

$$\Delta y = \frac{\lambda D}{d}$$

- Δy depends on :
  - i) the wavelength of light, λ
  - ii) the distance apart, d of the double slits,
  - iii) distance between slits and the screen, D
- Explanation for the above factors:
  - (a) if λ is short and hence Δy decreases for fixed D and d. The interference fringes are closer to each other and vice-versa.
  - (b) if the distance apart d of the slits diminished, Δy increased for fixed D and λ and vice-versa.

(c) if D increases Δy also increases for fixed λ and vice-versa.

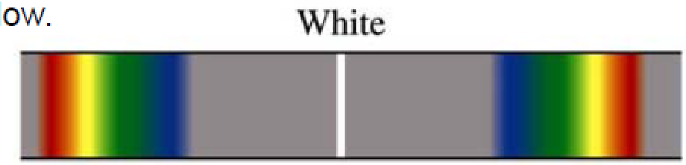
(d) if the source slit S_O is widened the fringes gradually disappear. The slit S_O then equivalent to large number of narrow slits, each producing its own fringe system at different places. The bright and dark fringes of different systems therefore overlap, giving rise to a different illumination.

(e) if one of the slit, S_1 or S_2 is covered up, the fringes disappear.

(f) if the source slit S_O is moved nearer the double slits, Δy is unaffected but their intensity increases.

(g) if the experiment is carried out in a different medium, for example water, the fringe separation Δy decreased or increased depending on the wavelength, λ of the medium.

(h) if white light is used the central bright fringe is white, and the fringes on either side are coloured. Blue is the colour nearer to the central fringe and red is farther away as shown in figure below.



26

Table below shows the range of wavelength for colours of visible light.

Colour	Range of λ/nm
Violet	400 – 450
Blue	450 – 520
Green	520 – 560
Yellow	560 – 600
Orange	600 – 625
Red	625 - 700

Example 7:

In a Young's double experiment, the slits separation is 1.0 mm. The distance between the slits and the screen is 1.0 m. The wavelength of the sodium light used is 5.9 x 10^{-5} cm.

a. Calculate the separation between two consecutive dark fringes.

b. If the sodium light is replaced with a blue light, what is the changes to the interference pattern on the screen?

<u>Solution</u>: $d = 1 \times 10^{-3}\ m,\ D = 1.0\ m,\ \lambda = 5.9 \times 10^{-7}\ m$

a. By applying the formula below,

$$\Delta y = \frac{\lambda D}{d}$$

$$\Delta y = 5.9x10^{-4}\,m$$

b. Sodium light is yellow

$$\lambda_{blue} < \lambda_{yellow} \quad \text{and} \quad \Delta y = \frac{\lambda D}{d},\ \text{where } D \text{ and } d \text{ are constant}$$

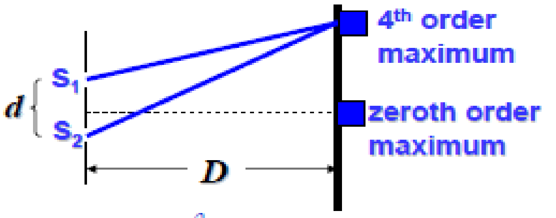
$$\Delta y_{blue} < \Delta y_{yellow}$$

hence, **the fringes get closer to each other.** 28

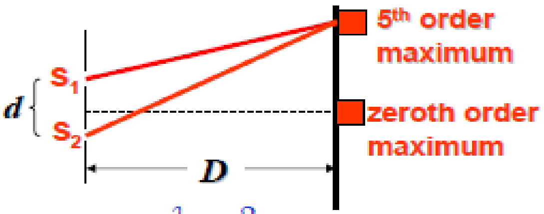
Example 8:

A monochromatic light of wavelength 600 nm falls on a system of double-slits of unknown slit separation. At the same time, the double-slits is illuminated by a monochromatic light of unknown wavelength. It was observed that the 4th order maximum of the known wavelength light overlapped with the 5th order maximum of the unknown wavelength light. Find the wavelength of the unknown wavelength light.

<u>Solution</u>: $\lambda_1 = 600 \times 10^{-9}\ m$



λ_1



$\lambda_2 = ?$

By applying the separation from central bright fringe for maximum (bright) fringe, thus

$$y_m = \frac{mD\lambda}{d}$$

For 4th order maximum :

$$y_4 = \frac{4D\lambda_1}{d}$$

For 5th order maximum :

$$y_5 = \frac{5D\lambda_2}{d}$$

Because the fringe is overlap, thus

$$y_4 = y_5$$

$$\frac{4D\lambda_1}{d} = \frac{5D\lambda_2}{d}$$

$$\lambda_2 = 4.8x10^{-7}\,m \quad or\ 480\,nm$$

Example 9: H.W

Young's double-slit experiment is performed with 589-nm light and a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

Ans: 1.54 *mm*

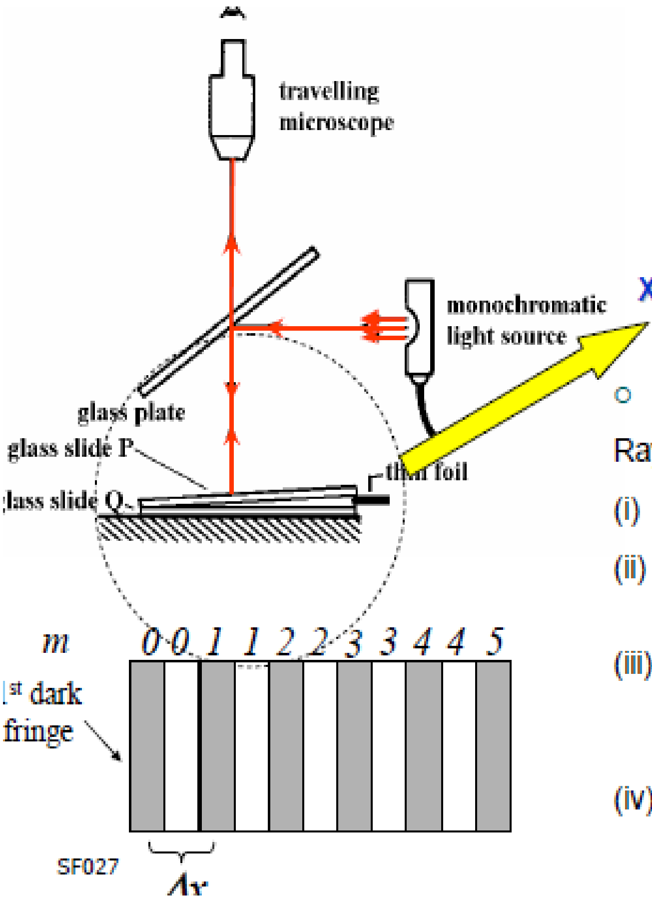
Example 10: H.W

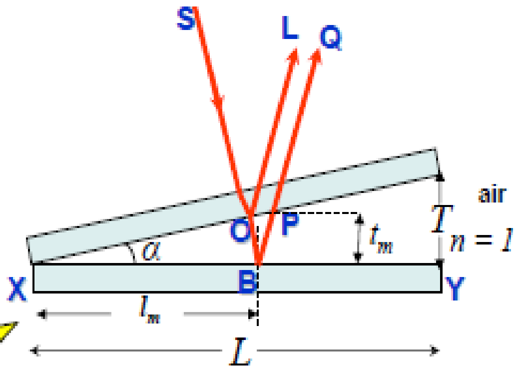
A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the centre of the pattern. What is the wavelength?

Ans: 515 *nm*

- S_1 and S_2 are two new sources of coherent waves in phase because they originate from the same source S_O .
- An interference pattern is formed on the screen.

11.4. Interference of Air Wedge





- Explanation:

Ray S incident at point O is

(i) partially reflected (ray OL)

(ii) refracted (OB) and then reflected at B (ray PQ)

(iii) The two wave-trains are coherent since both have originated from the same point O.

(iv) OL and PQ produce interference if brought together by the eye or a microscope

33

(v) Since the incidence is nearly normal (point P very close to O), the path difference between the rays at O (ray OL and ray OBPQ) is given by,

path difference, ΔL = OB +BP = $nt_m + nt_m$

= $2nt_m$ where n is refractive index of air = 1.0

(vi) At X, t_m = 0 and thus the path difference = 0 and a bright fringe is expected, but a dark fringe is observed at X. This is due to the phase change of π rad for ray PQ (reflected at a denser medium at B)

(vii)Hence, ray PQ is in antiphase with ray OL and when brought together (by the retina or lens) to interfere, a dark fringe is obtained.

For constructive interference : $2nt_m = m\lambda + \frac{1}{2}\lambda$
(bright fringe)

$$\boxed{2nt_m = (m+\frac{1}{2})\lambda; \quad m = 0,1,2,3,........} \quad .....(1)$$

For destructive interference :
(dark fringe)

$$\boxed{2nt_m = m\lambda; \quad m = 0,1,2,3,......} \quad .....(2)$$

- a phase change of π **rad** is equivalent to a path difference of ½ λ

- From equation (1) :

When $m = 0$, $t_0 = \frac{1}{4}\lambda$ ⟹ 1st bright fringe (1st order maximum)
$m = 1$, $t_1 = \frac{3}{4}\lambda$ ⟹ 2nd bright fringe (2nd order maximum)
$m = 2$, $t_2 = \frac{5}{4}\lambda$ ⟹ 3rd bright fringe (3rd order maximum)

i.e. Bright fringes are formed when the thickness of air film, $t_m = \frac{1}{4}\lambda, \frac{3}{4}\lambda, \frac{5}{4}\lambda, ...$

From equation (2) :

When $m = 0$, $t_0 = 0$ ⟹ 1st dark fringe (1st order minimum)
$m = 1$, $t_1 = \frac{1}{2}\lambda$ ⟹ 2nd dark fringe (2nd order minimum)
$m = 2$, $t_2 = \lambda$ ⟹ 3rd dark fringe (3rd order minimum)

i.e. Dark fringes are formed when the thickness of air film, $t_m = 0, \frac{1}{2}\lambda, \lambda, ,.......$

- Notes :
  - Ray reflected on a denser medium ⟹ **π rad phase change**
  - Ray reflected on a less dense medium ⟹ **no phase change** 35

The **separation** between the **1st dark fringe** to the **m th dark fringe**, l_m

$$\tan\alpha = \frac{T}{L} = \frac{t_m}{l_m}$$

$$l_m = \frac{t_m}{\tan\alpha} \quad .....(3)$$

From equation (2) : $t_m = \dfrac{m\lambda}{2n}$ substitute into equation (3)

$$\boxed{l_m = \frac{m\lambda}{2n\tan\alpha}} \quad .....(4)$$

where

$m = 0,1,2,.....$

λ : wavelength

n : refractive index

α : angle of inclination of glass slide 36

The **separation** between the **1st dark fringe** to the **m th bright fringe**, l_m

From equation (1) : $t_m = \dfrac{(m+\frac{1}{2})\lambda}{2n}$ substitute in equation (3)

$$\boxed{l_m = \frac{(m+\frac{1}{2})\lambda}{2n\tan\alpha}}$$

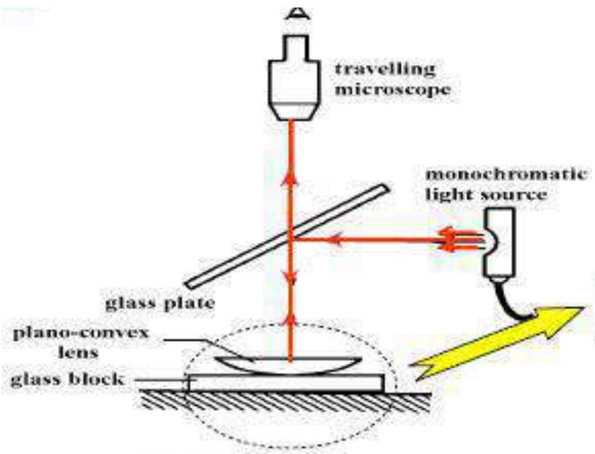
where $m = 0,1,2,.....$

The **separation** between **adjacent dark fringes or bright fringes**, Δx

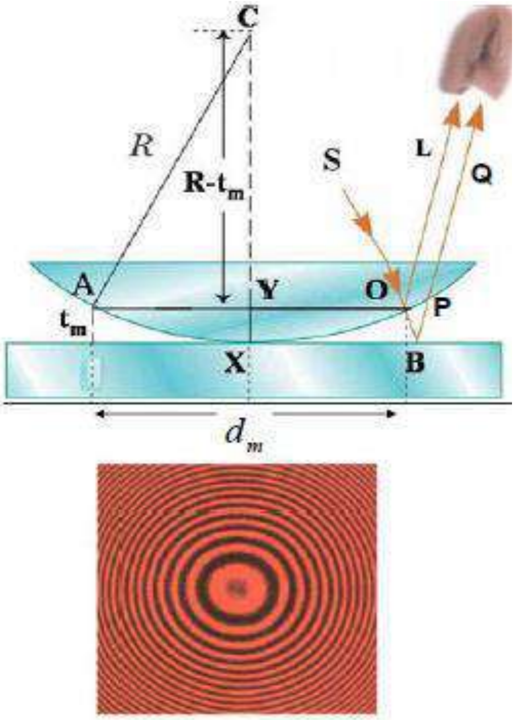
Put $m = 1$ into equation (4),

$$\boxed{\Delta x = \frac{\lambda}{2n\tan\alpha}}$$

12.4. Newton's Rings



(a) Apparatus setup



(b) Newton's Ring

Explanation for the dark circle at the centre is the same arguments as for air wedge.

SF027 38

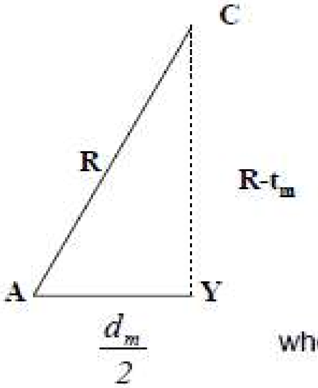
For constructive interference : (bright ring)

$$\boxed{2nt_m = (m+\frac{1}{2})\lambda; \quad m = 0,1,2,3,........} \quad .....(1)$$

For destructive interference : (dark ring)

$$\boxed{2nt_m = m\lambda; \qquad m = 0,1,2,3,......} \quad .....(2)$$

By using the figure below and Pythagoras' theorem:



$$R^2 = (R-t_m)^2 + \left(\frac{d_m}{2}\right)^2$$

$$R^2 = R^2 - 2Rt_m - t_m{}^2 + \frac{d_m{}^2}{4}$$

t_m is very small, hence $t_m{}^2 \approx 0$

$$\frac{d_m{}^2}{4} = 2Rt_m \quad .....(3)$$

where d_m : diameter of ring

R : radius of curvature

t_m : thickness of air

27 39

Diameter of dark ring

From equation (2) : $t_m = \frac{m\lambda}{2n}$ substitute in equation (3)

$$\frac{d_m^2}{4} = 2R\left(\frac{m\lambda}{2n}\right)$$

$$\boxed{d_m^2 = \frac{4Rm\lambda}{n}}$$ where $m = 0,1,2,.....$

When $m = 0 \Rightarrow$ Central dark spot (zeroth order minimum), $d_0 = 0$

$m = 1 \Rightarrow$ 1st dark ring (1st order minimum)

$m = 2 \Rightarrow$ 2nd dark ring (2nd order minimum)

$m = 3 \Rightarrow$ 3rd dark ring (3rd order minimum)

Diameter of bright ring

From equation (1) : $t_m = \frac{(m+\frac{1}{2})\lambda}{2n}$ substitute in equation (3)

$$\frac{d_m^{\ 2}}{4} = 2R\left[\frac{(m+\frac{1}{2})\lambda}{2n}\right]$$

$$\boxed{d_m^2 = \frac{4R(m+\frac{1}{2})\,\lambda}{n}}$$ where $m = 0,1,2,.....$

When $m = 0 \Rightarrow$ 1st bright ring (1st order maximum)

$m = 1 \Rightarrow$ 2nd bright ring (2nd order maximum)

$m = 2 \Rightarrow$ 3rd bright ring (3rd order maximum)

$m = 3 \Rightarrow$ 4th bright ring (4th order maximum)

Example 11:

In a Newton's ring experiment, the diameter of the m^{th} dark ring is 0.56 cm and the diameter of the (m+19)th dark ring is 1.34 cm. Determine the radius of curvature of the plano-convex used in the experiment if the wavelength of light used is 589 nm.

Solution: $d_m = 0.56 \text{ x } 10^{-2}$ m, $d_{m+19} = 1.34 \text{ x } 10^{-2}$ m, $\lambda = 589 \text{ x } 10^{-9}$ m , $n = 1$

For m^{th} dark ring : $d_m^2 = \frac{4Rm\lambda}{n}$

$$\left(0.56 \text{ x } 10^{-2}\right)^2 = 4Rm\lambda \qquad ....(1)$$

For (m+19)th dark ring : $d_{m+19}^2 = 4R(m+19)\lambda$

$$\left(1.34 \times 10^{-2}\right)^2 = 4R(m+19)\lambda \quad \ldots\ldots\ldots(2)$$

Divide eq. (2) by eq. (1): $\left(\frac{1.34}{0.56}\right)^2 = \frac{m+19}{m}$

$$5.73m - m = 19$$

$$m = 4 \quad \ldots\ldots\ldots(3)$$

Substitute eq. (3) into eq. (1):

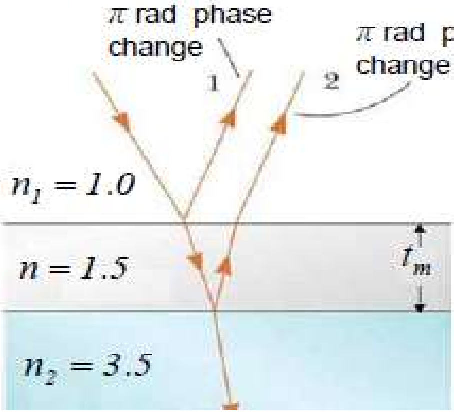
$$R = 3.33\ m$$

13.4. Thin Film Interference

- Thin oil films on water and soap bubble display rainbow like colours.
- The colours are the result of constructive interference between light reflected from two surfaces of the thin film.

Thin film on a denser medium, e.g non-reflective coating

- The figure below shows the interference between light waves reflected from the upper and lower surfaces of thin film (refractive index, n) on a denser medium.



Phase difference between ray 1 and ray 2,

$\Delta\phi = \pi - \pi = 0 \Longrightarrow$ **2 sources in phase**

Path difference between ray 1 and ray 2,

$$\text{path difference} = 2nt_m$$

Constructive interference:

$$\boxed{2nt_m = m\lambda}$$ where $m = 0,1,2,\ldots\ldots$

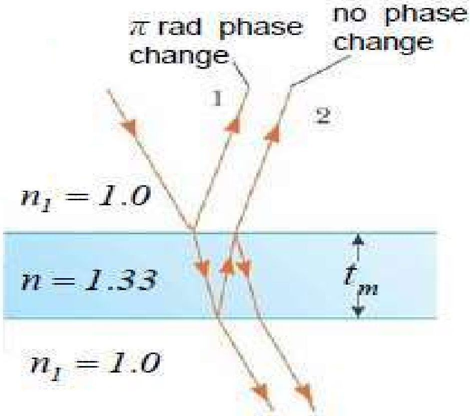
Destructive interference:

$$\boxed{2nt_m = (m+\tfrac{1}{2})\lambda}$$ where $m = 0,1,2,\ldots\ldots$

42

Thin film in air

- The figure below shows the interference between light waves reflected from the upper and lower surfaces of thin film (refractive index, n) in air.



Phase difference between ray 1 and ray 2,

$\Delta\phi = \pi - 0 = \pi \Longrightarrow$ **2 sources in antiphase**

Path difference between ray 1 and ray 2,

$$\text{path difference} = 2nt_m$$

Constructive interference:

$$\boxed{2nt_m = (m+\tfrac{1}{2})\lambda}$$ where $m = 0,1,2,\ldots$

Destructive interference:

$$\boxed{2nt_m = m\lambda}$$ where $m = 0,1,2,\ldots\ldots$

Example 12:

Two flat microscope slides of 45 mm in length are in contact along one edge, and the opposite edges are separated by a fine piece of wire. When the air-wedge formed is illuminated by light of wavelength 430 nm, interference fringes of separation 0.19 mm are observed.

Calculate:

a. the angle of the air wedge,

b. the diameter of the wire.

Solution: $\lambda = 430x10^{-9}\ m,\ L = 45x10^{-3}\ m,\ \Delta x = 0.19x10^{-3}\ m,\ n = 1$

a. By applying the equation of separation between fringes in air wedge,

$$\Delta x = \frac{\lambda}{2n\tan\alpha}$$

$$\alpha = \tan^{-1}\left(\frac{\lambda}{2n\Delta x}\right)$$

$$\alpha = 1.13\ x\ 10^{-3}\ rad \quad \text{or } 0.065°$$

b. From the figure, $\tan\alpha = \frac{d}{L}$

$$d = 5.09x10^{-5}\ m$$

Example 13:

A soap film reflects strongly red light (λ_1 = 700 nm) and green light (λ_2 = 500 nm) when illuminated by white light. If the refractive index of soap is 1.40, calculate minimum thickness of the soap film.

Solution: $n = 1.40$

For red light (λ_1) – Constructive interference occurs if

$$2nt_m = (m + \frac{1}{2})\lambda_1; \qquad m = 0,1,2,3,\ldots\ldots$$

$$t_m = \frac{(m+\frac{1}{2})\lambda_1}{2n}$$

When $m = 0$, hence

$$t_m = \frac{\lambda_1}{4n} = 125nm$$

For red light, the minimum thickness : $125nm$.

For green light (λ_2) – Constructive interference occurs if

$$t_m = \frac{(m+\frac{1}{2})\lambda_2}{2n}$$

When $m = 0$, hence

$$t_m = \frac{\lambda_2}{4n} = 89.3\ nm$$

For green light, the minimum thickness : $89.3\ nm$.