1.1. Refraction

- Definition – is defined as *the changing of direction of a light ray and its speed of propagation as it passes from one medium into another.*
- Laws of refraction state :
  - The incident ray, the refracted ray and the normal all lie in the same plane.
  - For two given media,

$$\boxed{\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \text{constant}}$$

Or

$$\boxed{n_1 \sin i = n_2 \sin r}$$

Snell's law

where

i : angle of incidence

r : angle of refraction

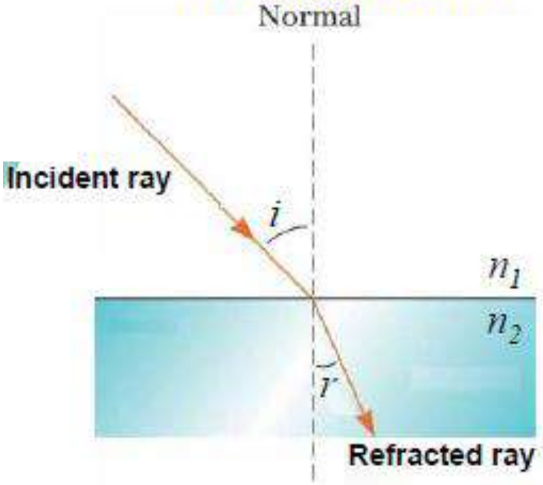
n_1 : refractive index of the medium 1 (Medium containing the incident ray)

n_2 : refractive index of the medium 2 (Medium containing the refracted ray)

- Examples for refraction of light ray travels from one medium to another medium can be shown in figures below.

(a) $n_1 < n_2$

(Medium 1 is less dense than medium 2)

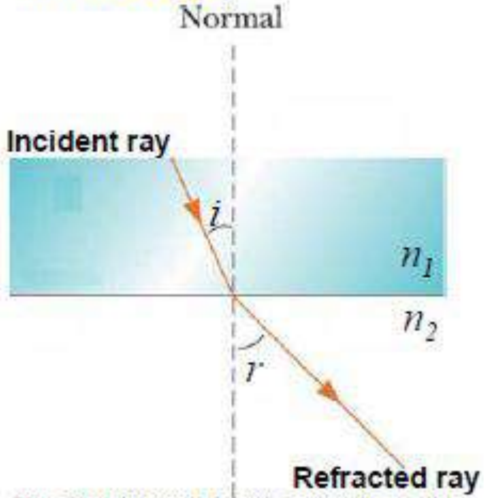


The light ray is bent toward the normal, thus

$r < i$

(b) $n_1 > n_2$

(Medium 1 is denser than medium 2)



The light ray is bent away from the normal, thus

$r > i$

- Refractive index (**index of refraction**)
  - Definition – is defined as *the constant ratio* $\dfrac{\sin i}{\sin r}$ *for the two given media.*
  - The value of refractive index depends on the type of medium and the colour of the light.
  - It is dimensionless and its value greater than 1.
  - Consider the light ray travels from medium 1 into medium 2, the refractive index can be denoted by

$$\boxed{{}_1n_2 = \frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}} = \frac{v_1}{v_2}}$$

(Medium containing the incident ray)

(Medium containing the refracted ray)

  - Absolute refractive index, n (for the incident ray is travelling in **vacuum or air** and is then refracted into the **medium concerned**) is given by

$$\boxed{n = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}} = \frac{c}{v}}$$

- The relationship between refractive index and the wavelength of light.
  - As light travels from one medium to another, its **wavelength, λ changes** but its **frequency, f remains constant**.
  - The wavelength changes because of **different material**. The frequency remains constant because the number of wave cycles arriving per unit time must equal the number leaving per unit time so that the boundary surface **cannot create or destroy waves**.
  - By considering a light travels from medium 1 (n_1) into medium 2 (n_2), the velocity of light in each medium is given by

$$v_1 = f\lambda_1 \quad \text{and} \quad v_2 = f\lambda_2$$

then

$$\frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} \quad \text{where} \quad v_1 = \frac{c}{n_1} \quad \text{and} \quad v_2 = \frac{c}{n_2}$$

$$\frac{\left(\dfrac{c}{n_1}\right)}{\left(\dfrac{c}{n_2}\right)} = \frac{\lambda_1}{\lambda_2} \quad \Longrightarrow \quad \boxed{n_1\lambda_1 = n_2\lambda_2}$$

(Refractive index is inversely proportional to the wavelength)

- If medium 1 is vacuum or air, then n_1 = 1. Hence the refractive index for any medium, n can be expressed as

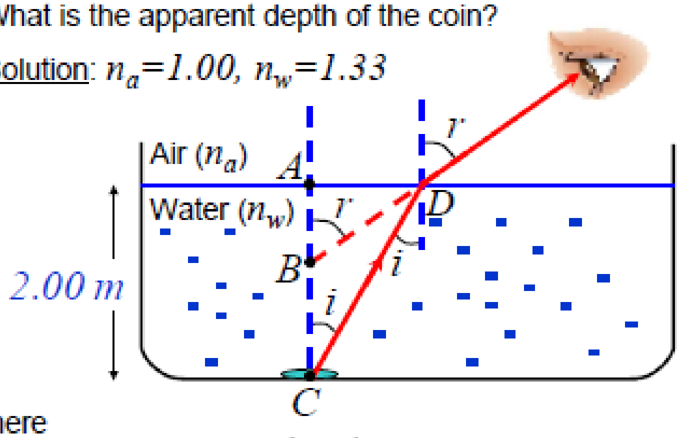
$$\boxed{n = \frac{\lambda_0}{\lambda}}$$

where
λ_0 : wavelength of light in vacuum
λ : wavelength of light in medium

- **Example 1 :**

A fifty cent coin is at the bottom of a swimming pool of depth 2.00 m. The refractive index of air and water are 1.00 and 1.33 respectively. What is the apparent depth of the coin?

Solution: $n_a=1.00,\ n_w=1.33$



where
AB : apparent depth
AC : actual depth = 2.00 m

From the diagram,

△ABD ⟹ $\tan r = \dfrac{AD}{AB}$

△ACD ⟹ $\tan i = \dfrac{AD}{AC}$

By considering only small angles of r and i , hence

$\tan r \approx \sin r$ and $\tan i \approx \sin i$

then

$$\frac{\tan i}{\tan r} = \frac{\sin i}{\sin r} = \frac{\left(\dfrac{AD}{AC}\right)}{\left(\dfrac{AD}{AB}\right)} = \frac{AB}{AC}$$

From the Snell's law,

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{n_a}{n_w}$$

$$\frac{AB}{AC} = \frac{n_a}{n_w}$$

$$AB = 1.50\ m$$

Note : (Important)

Other equation for absolute refractive index in term of depth is given by

$$n = \frac{\text{real depth}}{\text{apparent depth}}$$

Example 2:

A light beam travels at 1.94×10^8 m s^{-1} in quartz. The wavelength of the light in quartz is 355 nm.

a. Find the index of refraction of quartz at this wavelength.

b. If this same light travels through air, what is its wavelength there?

(Given the speed of light in vacuum, $c = 3.00 \times 10^8$ m s^{-1})

No. 33.3, pg. 1278, University Physics with Modern Physics, 11th edition, Young & Freedman.

Solution: $v = 1.94 \times 10^8\ m\ s^{-1}$, $\lambda = 355 \times 10^{-9}\ m$

a. By applying the equation of absolute refractive index, hence

$$n = \frac{c}{v}$$

$$n = 1.55$$

b. By using the equation below, thus

$$n = \frac{\lambda_0}{\lambda}$$

$$\lambda_0 = n\lambda$$

$$\lambda_0 = 5.50x10^{-7}\ m\ @\ 550\ nm$$

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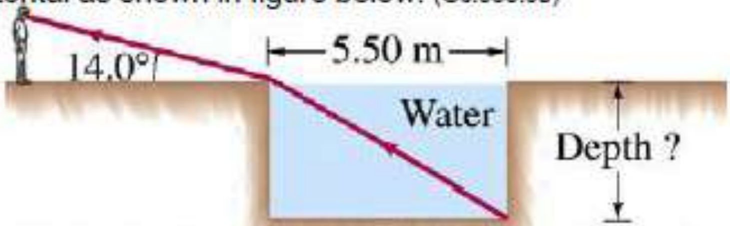
Example 3 : (H.W)

We wish to determine the depth of a swimming pool filled with water by measuring the width (x = 5.50 m) and then noting that the bottom edge of the pool is just visible at an angle of 14.0° above the horizontal as shown in figure below. (Gc.835.60)

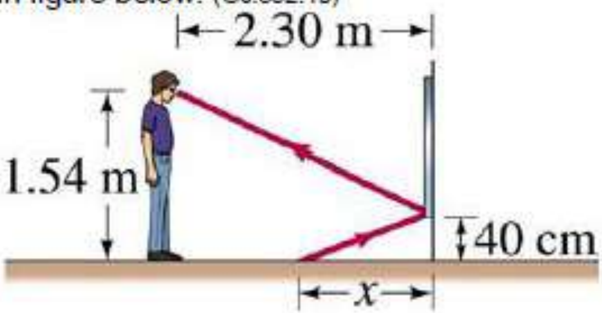


Calculate the depth of the pool. (Given $n_{water} = 1.33$ and $n_{air} = 1.00$)

Ans. : 5.16 m

Example 4 : (H.W)

A person whose eyes are 1.54 m above the floor stands 2.30 m in front of a vertical plane mirror whose bottom edge is 40 cm above the floor as shown in figure below. (Gc.832.10)



Find x.

Ans. : 0.81 m

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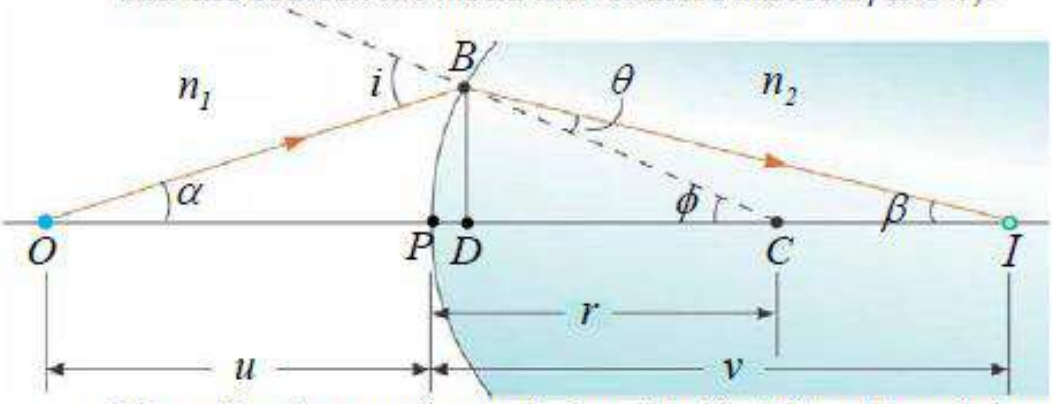
1.2. Refraction of Spherical Surfaces

- Figure below shows a spherical surface with radius, r forms an interface between two media with refractive indices n_1 and n_2.



- The surface forms an image I of a point object O as shown in figure above.
- The incident ray OB making an angle i with the normal and is refracted to ray BI making an angle θ where $n_1 < n_2$.
- Point C is the centre of curvature of the spherical surface and BC is normal.

- From the figure,

$$\triangle BOC \Longrightarrow i = \alpha + \phi \quad \text{------(1)}$$

$$\triangle BIC \Longrightarrow \phi = \beta + \theta$$

$$\theta = \phi - \beta \quad \text{------(2)}$$

- From the Snell's law

$$n_1 \sin i = n_2 \sin \theta$$

By using △BOD, △BCD and △BID thus

$$\tan\alpha = \frac{BD}{OD}\,;\ \tan\phi = \frac{BD}{CD}\,;\ \tan\beta = \frac{BD}{ID}$$

By considering point B very close to the pole P, hence

$$\sin i \approx i\,;\ \sin\theta \approx \theta\,;\ \tan\alpha \approx \alpha\,;\ \tan\phi \approx \phi\,;\ \tan\beta \approx \beta$$

$$OD \approx OP = u\,;\ CD \approx CP = r\,;\ ID \approx IP = v$$

then Snell's law can be written as

$$n_1 i = n_2 \theta \quad \text{------(3)}$$

- By substituting eq. (1) and (2) into eq. (3), thus

$$n_1(\alpha + \phi) = n_2(\phi - \beta)$$

$$n_1\alpha + n_2\beta = (n_2 - n_1)\phi$$

then

$$n_1\left(\frac{BD}{u}\right) + n_2\left(\frac{BD}{v}\right) = (n_2 - n_1)\left(\frac{BD}{r}\right)$$

$$\boxed{\frac{n_1}{u}+\frac{n_2}{v}=\frac{(n_2-n_1)}{r}}$$

Equation of spherical refracting surface

where

v : image distance from pole
u : object distance from pole
n_1 : refractive index of medium 1
(Medium containing the incident ray)
n_2 : refractive index of medium 2
(Medium containing the refracted ray)

- Note :
  - If the refraction surface is **flat (plane)** :

    $r=\infty$ then $\boxed{\frac{n_1}{u}+\frac{n_2}{v}=0}$

  - The equation (formula) of linear magnification for refraction by the spherical surface is given by

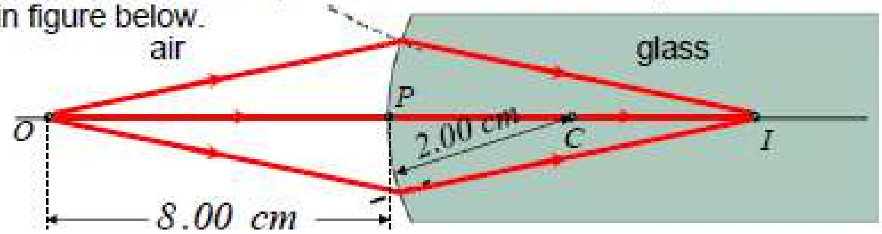
$$\boxed{M=\frac{h_i}{h_o}=-\frac{n_1 v}{n_2 u}}$$

- Sign convention for **refraction** :

Physical Quantity	Positive sign (+)	Negative sign (-)
Object distance, u	Real object (in front of the refracting surface)	Virtual object (at the back of the refracting surface)
Image distance, v	Real image (opposite side of the object)	Virtual image (same side of the object)
Focal length, f	Convex surface	Concave surface
Radius of curvature, r	Convex surface	Concave surface
Linear magnification, M	Upright (erect) image	Inverted image

Example 5 :

A cylindrical glass rod in air has refractive index of 1.52. One end is ground to a hemispherical surface with radius, r =2.00 cm as shown in figure below.



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Find,

a. the position of the image for a small object on the axis of the rod, 8.00 cm to the left of the pole as shown in figure.

b. the linear magnification.

(Given the refractive index of air , n_a= 1.00)

Example 34.5, pg. 1302, University Physics with Modern Physics,11th edition, Young & Freedman.

<u>Solution</u>: $n_g=1.52,\ u=8.00\ cm,\ r=+2.00\ cm$

a. By applying the equation of spherical refracting surface,

$$\frac{n_1}{u}+\frac{n_2}{v}=\frac{(n_2-n_1)}{r}$$

$$\frac{n_a}{u}+\frac{n_g}{v}=\frac{(n_g-n_a)}{r}$$

$$v=+11.26\ cm$$

The image is 11.26 cm at the back of the convex surface.

b. By using the equation of linear magnification for refracting surface,

$$M=\frac{h_i}{h_o}=-\frac{n_1 v}{n_2 u} \Rightarrow M=-\frac{n_a v}{n_g u}$$

$$M=-0.93$$

Negative sign indicate the image is inverted.

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Example 6 : (H.W)

A small strip of paper is pasted on one side of a glass sphere of radius 5 cm. The paper is then view from the opposite surface of the sphere. Find the position of the image.

(Given refractive index of glass =1.52 and refractive index of air=1.00)

Ans. : 20.83 cm in front of the concave surface (second refracting surface)

Example 7 : (H.W)

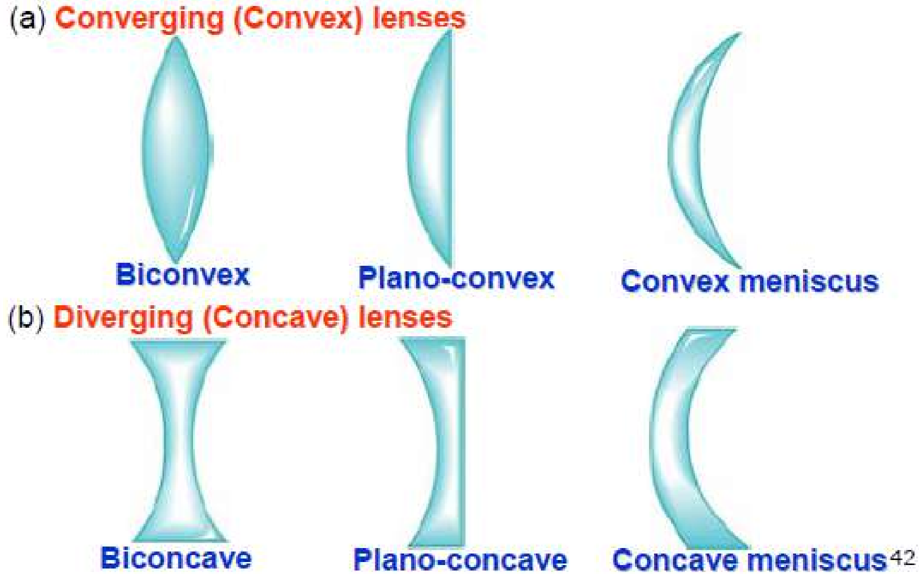
A point source of light is placed at a distance of 25.0 cm from the centre of a glass sphere of radius 10 cm. Find the image position of the source. (Gc.830.Exam.33-11)

(Given refractive index of glass =1.50 and refractive index of air=1.00)

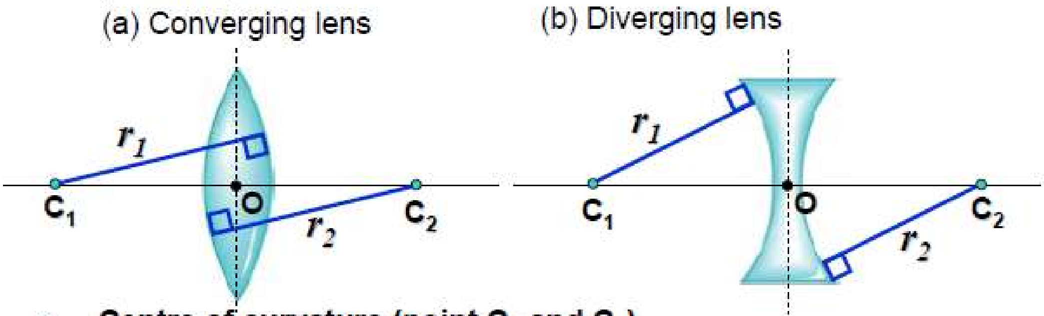
Ans. : 28 cm at the back of the concave surface (second refracting surface).

1.3.Thin Lens

- Definition – is defined as *a transparent material with two spherical refracting surfaces whose thickness is thin compared to the radii of curvature of the two refracting surfaces.*
- There are two types of thin lens. It is **converging** and **diverging** lens.
- Figures below show the various types of thin lenses, both converging and diverging.



- Figures below show the shape of converging (convex) and diverging (concave) lenses.



- **Centre of curvature (point C_1 and C_2)**
  - is defined as *the centre of the sphere of which the surface of the lens is a part.*
- **Radius of curvature (r_1 and r_2)**
  - is defined as *the radius of the sphere of which the surface of the lens is a part.*
- **Principal (Optical) axis**
  - is defined as *the line joining the two centres of curvature of a lens.*
- **Optical centre (point O)**
  - is defined as *the point at which any rays entering the lens pass without deviation.*

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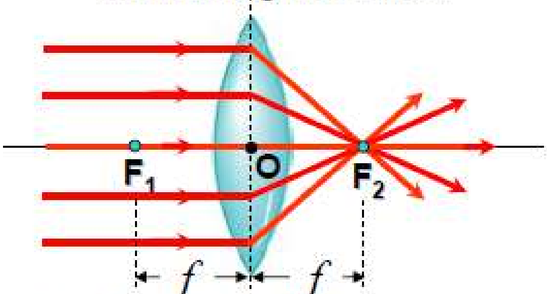
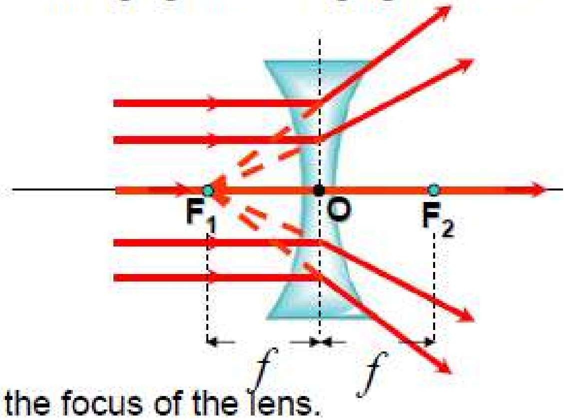
- Consider the ray diagrams for converging and diverging lens as shown in figures below.

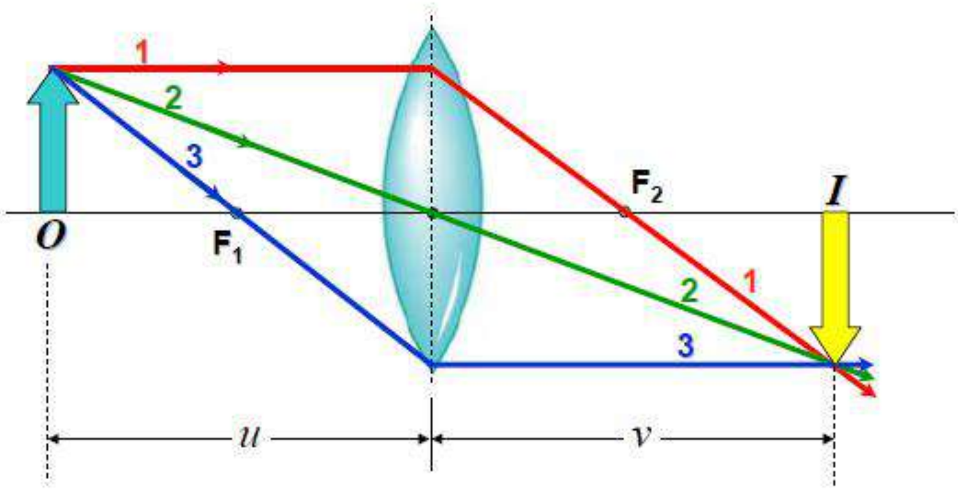


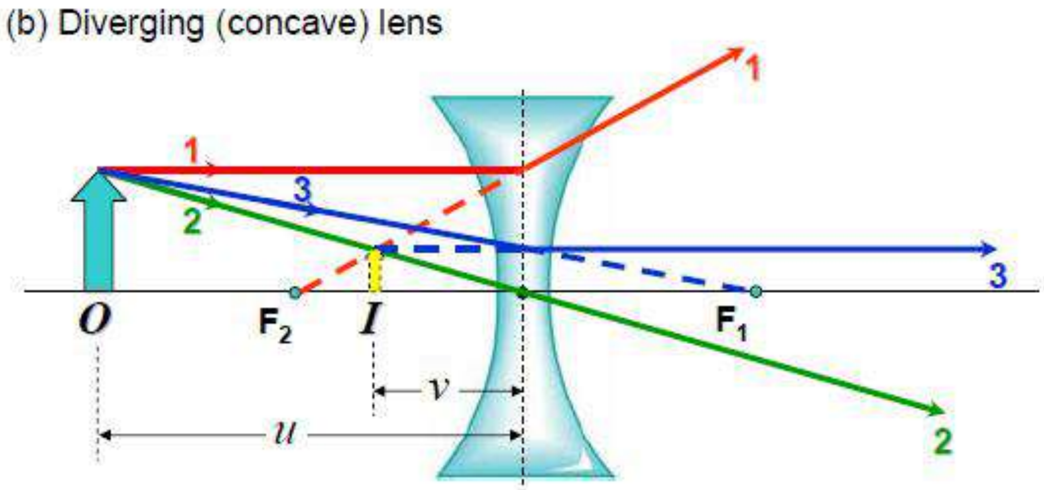


- From the figures,
  - Point F_1 and F_2 represent the focus of the lens.
  - Distance f represents the focal length of the lens.
- **Focus (point F_1 and F_2)**
  - For **converging (convex)** lens – is defined as *the point on the principal axis where rays which are parallel and close to the principal axis converges after passing through the lens.*
    - Its focus is real (principal).
  - For **diverging (concave)** lens – is defined as *the point on the principal axis where rays which are parallel to the principal axis seem to diverge from after passing through the lens.*
    - Its focus is virtual.
- **Focal length (f)**
  - Definition – is defined as *the distance between the focus F and the optical centre O of the lens.*

1.4.Ray Diagrams For Thin Lenses

- Ray diagrams below showing the graphical method of locating an image formed by converging (convex) and diverging (concave) lenses
  (a) Converging (convex) lens



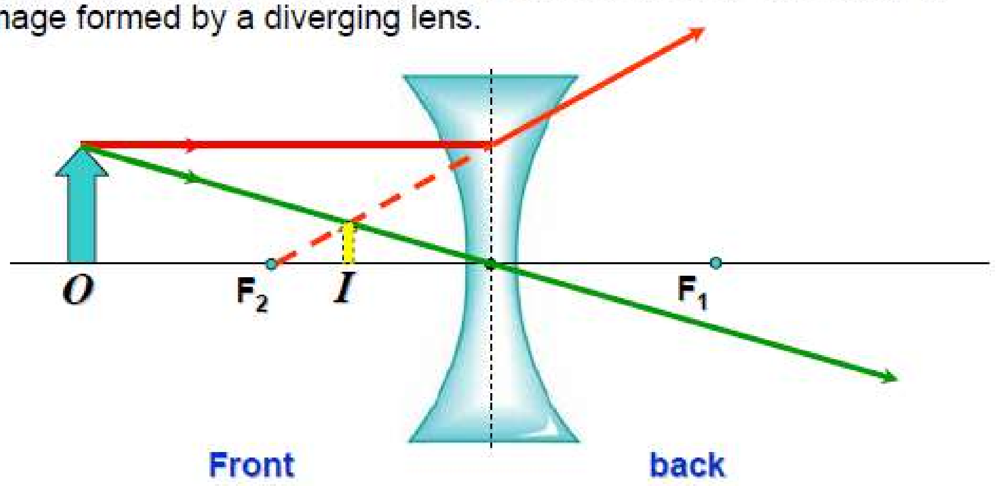


At least any two rays for drawing the ray diagram.

- **Ray 1** - Parallel to the principal axis, after refraction by the lens, passes through the focal point (focus) F_2 of a converging lens or appears to come from the focal point F_2 of a diverging lens.
- **Ray 2** - Passes through the optical centre of the lens is undeviated.
- **Ray 3** - Passes through the focus F_1 of a converging lens or appears to converge towards the focus F_1 of a diverging lens, after refraction by the lens the ray parallel to the principal axis.

1.5.Images Formed by a diverging Lens

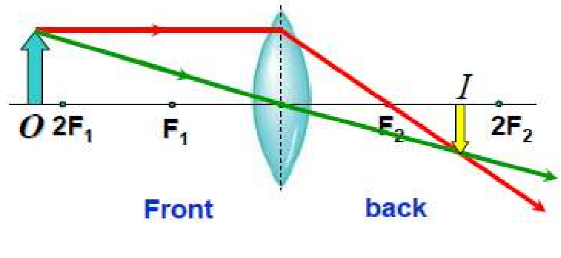
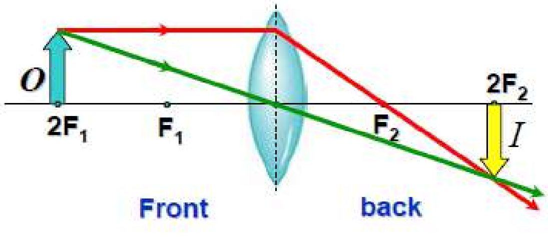
- Ray diagrams below showing the graphical method of locating an image formed by a diverging lens.

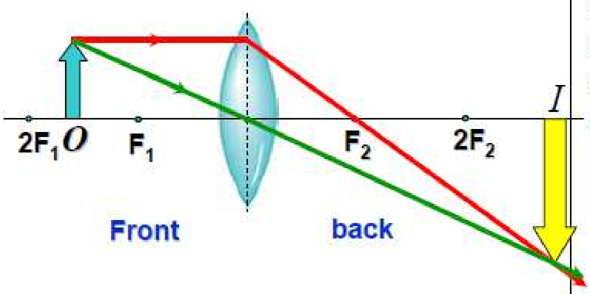
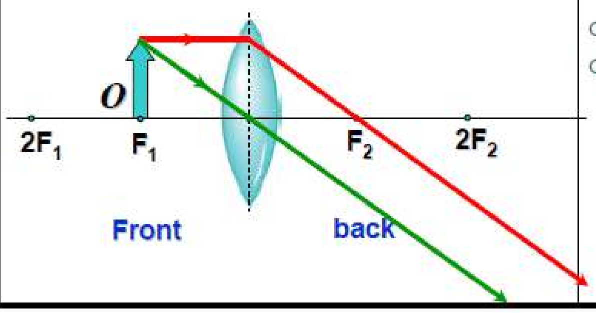


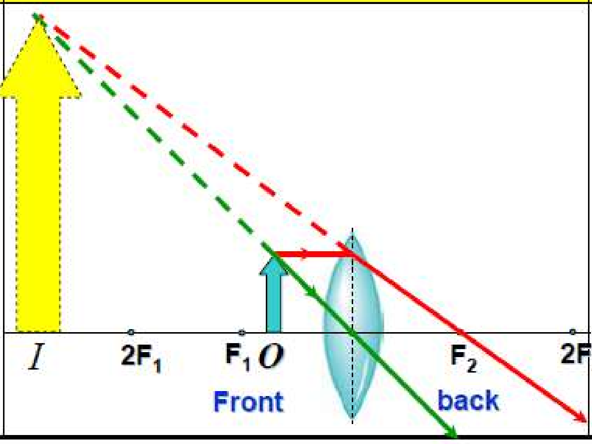
- Properties of image formed are
  - virtual
  - upright
  - diminished (smaller than the object)
  - formed in front of the lens.
- Object position $\rightarrow$ any position in front of the diverging lens.

1.6.Images Formed by a converging Lens

- Table below shows the ray diagrams of locating an image formed by a converging lens for various object distance, u.

Object distance, u	Ray diagram	Image property
$u > 2f$		○ Real ○ Inverted ○ Diminished ○ Formed between point F_2 and $2F_2$. (at the back of the lens)
$u = 2f$		○ Real ○ Inverted ○ Same size ○ Formed at point $2F_2$. (at the back of the lens)

Object distance, u	Ray diagram	Image property
$f < u < 2f$		○ Real ○ Inverted ○ Magnified ○ Formed at a distance greater than $2f$ at the back of the lens.
$u = f$		○ Real ○ Formed at infinity.

Object distance, u	Ray diagram	Image property
$u < f$		Virtual Upright Magnified Formed in front of the lens.

- Linear (lateral) magnification of the thin lenses, M is defined as *the ratio between image height, h_i and object height, h_o*

Simulation

$$\boxed{M = \frac{h_i}{h_o} = -\frac{v}{u}}$$

where
v : image distance from optical centre
u : object distance from optical centre

Negative sign indicates that when u and v are both positive, the image is inverted and h_o and h_i have opposite signs.

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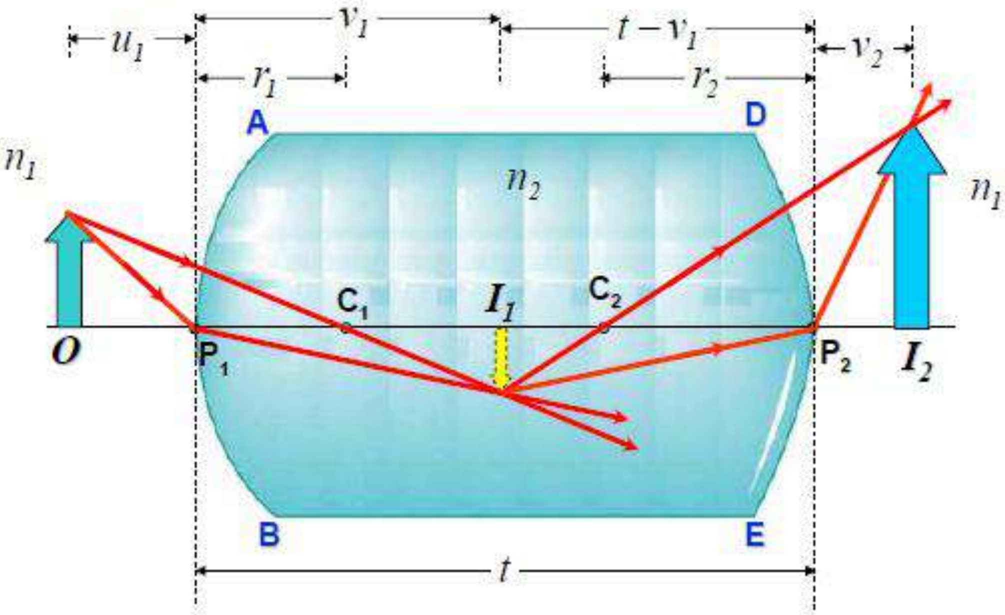
1.7.Thin Lens Formula And Lens Maker's Equation

- Considering the ray diagram of refraction for 2 spherical surfaces as shown in figure below.



- By using the equation of spherical refracting surface, the refraction by first surface AB and second surface DE are given by
  - **Convex surface AB ($r = +r_1$)**

$$\frac{n_1}{u_1}+\frac{n_2}{v_1}=\frac{(n_2-n_1)}{r_1} \quad \text{------------(1)}$$

  - **Concave surface DE ($r = -r_2$)**

$$\frac{n_2}{(t-v_1)}+\frac{n_1}{v_2}=\frac{(n_1-n_2)}{-r_2}$$

Assuming the lens is very thin thus $t = 0$,

$$\frac{n_2}{-v_1}+\frac{n_1}{v_2}=\frac{(n_1-n_2)}{-r_2}$$

$$\frac{n_2}{v_1}=-\left[\left(\frac{n_1-n_2}{-r_2}\right)-\frac{n_1}{v_2}\right]$$

$$\frac{n_2}{v_1}=\frac{n_1}{v_2}-\left(\frac{n_2-n_1}{r_2}\right) \quad \text{------------(2)}$$

- By substituting eq. (2) into eq. (1), thus

$$\frac{n_1}{u_1}+\left[\frac{n_1}{v_2}-\left(\frac{n_2-n_1}{r_2}\right)\right]=\frac{(n_2-n_1)}{r_1}$$

$$\frac{n_1}{u_1}+\frac{n_1}{v_2}=\frac{(n_2-n_1)}{r_1}+\frac{(n_2-n_1)}{r_2}$$

then

$$\frac{1}{u_1}+\frac{1}{v_2}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right) \quad \text{------------(3)}$$

- If $u_1 = \infty$ and $v_2 = f$ hence eq. (3) becomes

$$\boxed{\frac{1}{f}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)} \Rightarrow$$

Lens maker's equation

where

f : focal length
r_1 : radius of curvature of first refracting surface
r_2 : radius of curvature of second refracting surface
n_1 : refractive index of the medium
n_2 : refractive index of the lens material

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- By equating eq. (3) with lens maker's equation, hence

$$\frac{1}{u_1}+\frac{1}{v_2}=\frac{1}{f}$$

therefore in general,

$$\boxed{\frac{1}{f}=\frac{1}{u}+\frac{1}{v}} \Rightarrow \text{Thin lens formula}$$

- Note :
  - If the medium is **air** ($n_1 = n_{air} = 1$) thus the lens maker's equation will be

$$\boxed{\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}$$

where n : refractive index of the lens material

  - For thin lens formula and lens maker's equation, Use the **sign convention** for **refraction**. → **Very Important**
  - The radius of curvature of flat refracting surface is infinity, $r = \infty$.

Example 8 :

A biconvex lens is made of glass with refractive index 1.52 having the radii of curvature of 20 cm respectively. Calculate the focal length of the lens in

a. water,

b. carbon disulfide.

(Given n_w = 1.33 and n_c=1.63)

Solution: $r_1=+20\ cm,\ r_2=+20\ cm,\ n_g=n_2=1.52$

a. Given the refractive index of water, $n_w = n_1$

By using the lens maker's equation, thus

$$\frac{1}{f}=\left(\frac{n_g}{n_w}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$f=+70\ cm$$

b. Given the refractive index of carbon disulfide, $n_c = n_1$

By using the lens maker's equation, thus

$$\frac{1}{f}=\left(\frac{n_g}{n_c}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$f=-148.18\ cm$$

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Example 9 :

A converging lens with a focal length of 90.0 cm forms an image of a 3.20 cm tall real object that is to the left of the lens. The image is 4.50 cm tall and inverted. Find

a. the object position from the lens.

b. the image position from the lens. Is the image real or virtual?

No. 34.26, pg. 1331, University Physics with Modern Physics,11th edition, Young & Freedman.

Solution: $f=+90.0\ cm,\ h_o=3.20\ cm,\ h_i=-4.50\ cm$

a. By using the linear magnification equation, hence

$$M=\frac{h_i}{h_o}=-\frac{v}{u}$$

$$v=1.41u \quad \text{------------(1)}$$

By applying the thin lens formula,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{90.0}=\frac{1}{u}+\frac{1}{v} \quad \text{------------(2)}$$

By substituting eq. (1) into eq. (2),hence

$$u=154\ cm$$

The object is placed 154 cm in front of the lens.

b. By substituting $u=154\ cm$ into eq. (1),therefore

$$v=217\ cm$$

The image forms 217 cm at the back of the lens (at the opposite side of the object placed) and the image is real.

Example 10 :

An object is placed 90.0 cm from a glass lens (n=1.56) with one concave surface of radius 22.0 cm and one convex surface of radius 18.5 cm. Determine

a. the image position.

b. the linear magnification. (Gc.862.28)

Solution: $u=+90.0\ cm,\ n=1.56,\ r_1=-22.0\ cm,\ r_2=+18.5\ cm$

a. By applying the lens maker's equation in air,

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$f=+208\ cm$$

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By applying the thin lens formula, thus

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$v = -159\ cm$$

The image forms 159 cm in front of the lens (at the same side of the object placed)

b. By applying equation of linear magnification for thin lens, thus

$$M = -\frac{v}{u} \Longrightarrow M = 1.77$$

Example (11): H.W

A glass (n=1.50) plano-concave lens has a focal length of 21.5 cm. Calculate the radius of the concave surface. (Gc.862.26)

Ans. : -10.8 cm

Example (12): H.W

An object is 16.0 cm to the left of a lens. The lens forms an image 36.0 cm to the right of the lens.

a. Calculate the focal length of the lens and state the type of the lens.

b. If the object is 8.00 mm tall, find the height of the image.

c. Sketch the ray diagram for the case above. (UP. 1332.34.34)

Ans. : +11.1 cm, -1.8 cm 58