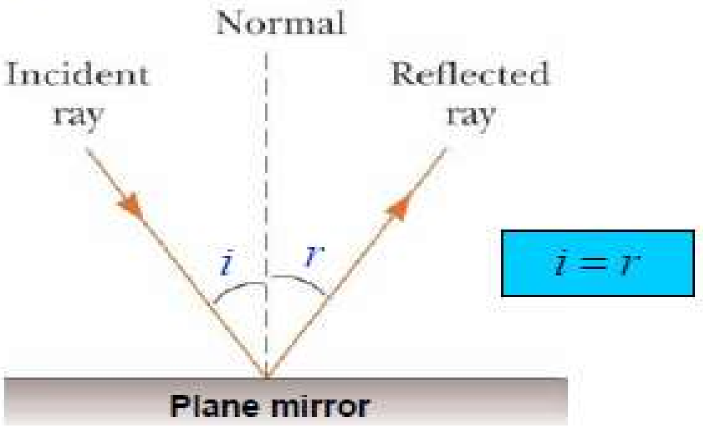
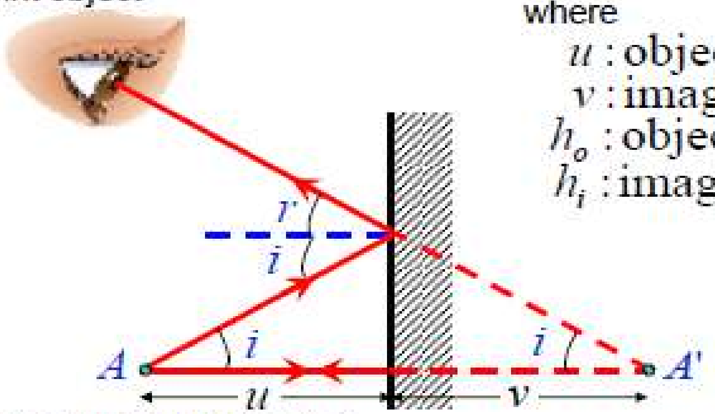
1.2. REFLECTION

- Definition – is defined as *the return of all or part of a beam of particles or waves when it encounters the boundary between two media.*
- Laws of reflection state :
  - The incident ray, the reflected ray and the normal all lie in the same plane.
  - The angle of incidence, i equals the angle of reflection, r as shown in figure below.

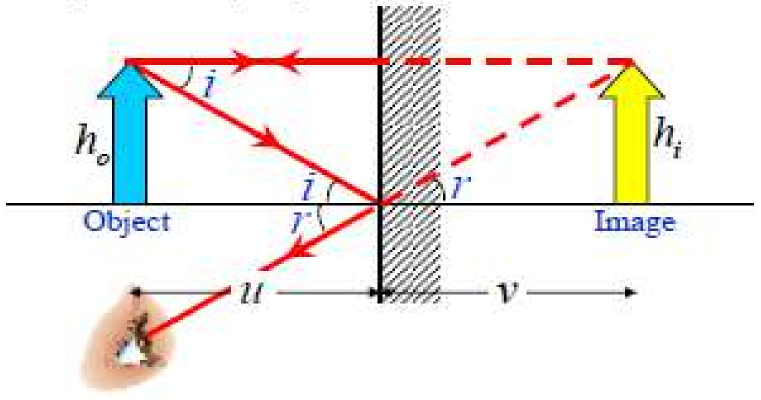


- Image formation by a plane mirror.
  - Point object



where
u : object distance
v : image distance
h_o : object height
h_i : image height

  - Vertical (extended) object



- The properties of image formed are
  - virtual
  - upright or erect
  - laterally reverse
  - the object distance, u equals the image distance, v
  - the same size where the linear magnification is given by

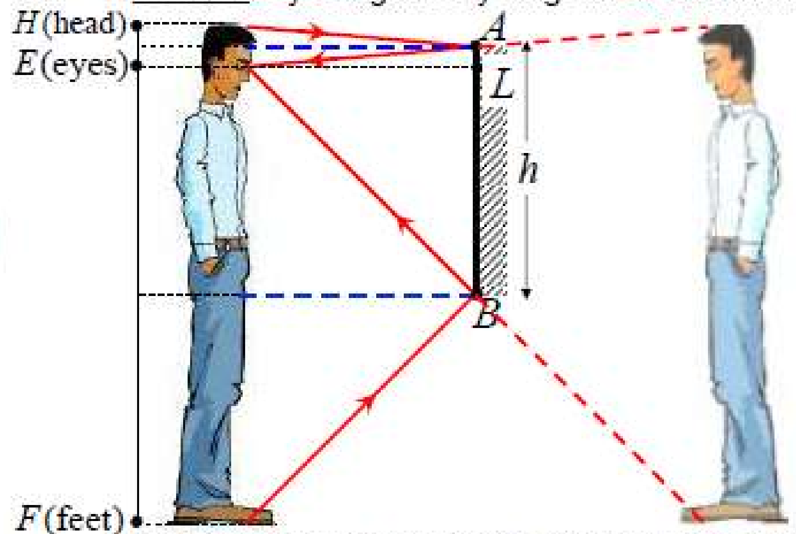
$$M = \frac{\text{Image height, } h_i}{\text{Object height, } h_o} = 1$$

  - obey the laws of reflection.

- **Example 1 :**

Find the minimum vertical length of a plane mirror for an observer of 2.0 m height standing upright close to the mirror to see his whole reflection. How should this minimum length mirror be placed on the

Solution: By using the ray diagram as shown in figure below.



$$AL = \frac{1}{2}HE$$

$$LB = \frac{1}{2}EF$$

The minimum vertical length of the mirror is given by

$$h = AL + LB$$

$$h = \frac{1}{2}HE + \frac{1}{2}EF$$

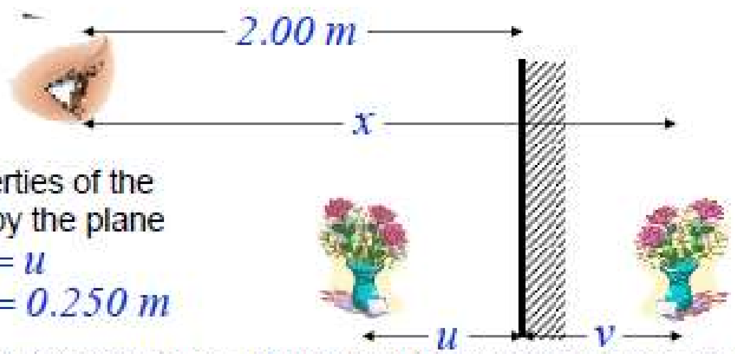
$$h = \frac{1}{2}\underbrace{(HE + EF)}_{\text{Height of observer}} \Rightarrow h = 1.0\ m$$

The mirror can be placed on the wall with the lower end of the mirror is halved of the distance between the eyes and feet of the observer.

- **Example 2 :**

A rose in a vase is placed 0.250 m in front of a plane mirror. Nagar looks into the mirror from 2.00 m in front of it. How far away from Nagar is the image of the rose?

Solution: $u = 0.250\ m$



From the properties of the image formed by the plane mirror, thus $v = u$

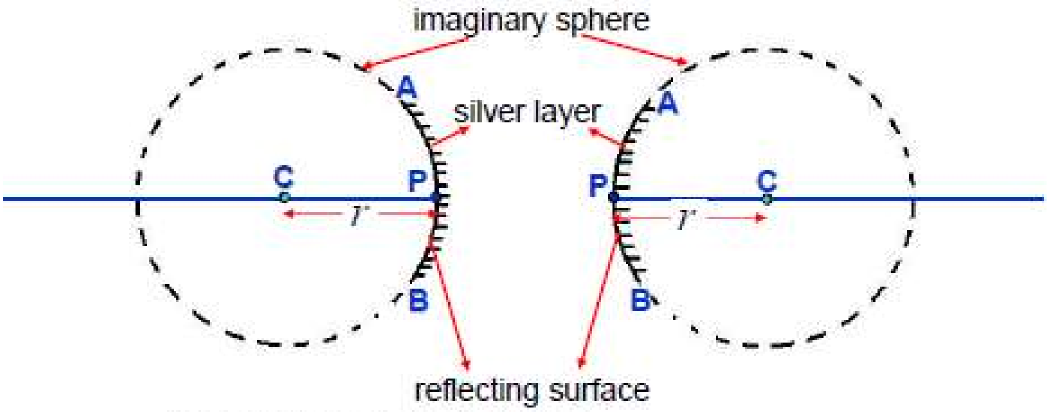
$$v = 0.250\ m$$

Therefore, the distance between Nagar and the image of the rose is given by $x = 2.00 + v \Longrightarrow x = 2.25\ m$

2.2. Reflection of Spherical Mirrors

- Definition – is defined as *a reflecting surface that is part of a sphere.*
- There are two types of spherical mirror. It is **convex** (curving outwards) and **concave** (curving inwards) mirror.
- Figures below show the shape of concave and convex mirrors.

(a) Concave (**Converging**) mirror (b) Convex (**Diverging**) mirror

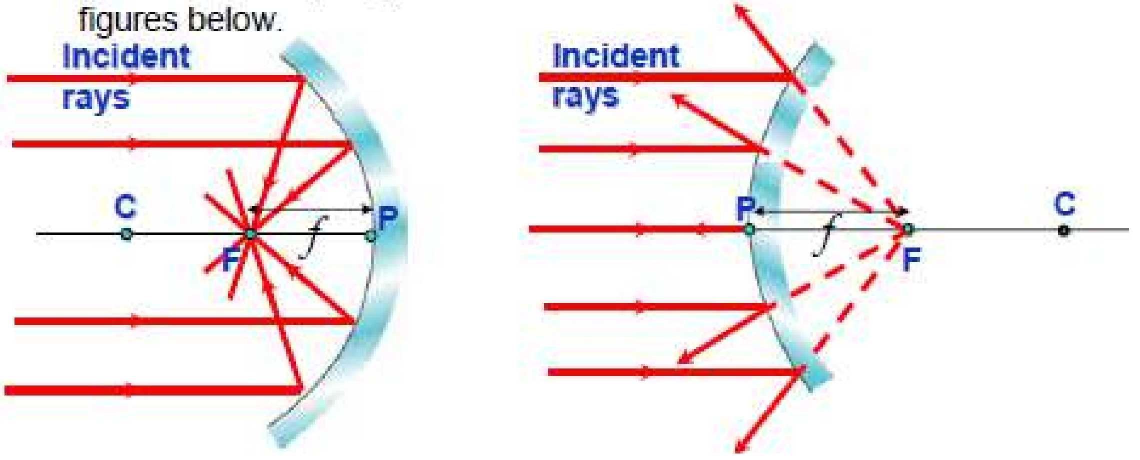


- Some terms of spherical mirror
  - **Centre of curvature (point C)**
    - is defined as *the centre of the sphere of which a curved mirror forms a part.*

- **Radius of curvature, *r***
  - is defined as *the radius of the sphere of which a curved mirror forms a part.*
- **Pole or vertex (point P)**
  - is defined as *the point at the centre of the mirror.*
- **Principal axis**
  - is defined as *the straight line through the centre of curvature C and pole P of the mirror.*
- AB is called the **aperture** of the mirror.

1.2.2.Focal point (F) and Focal length (*f*) :

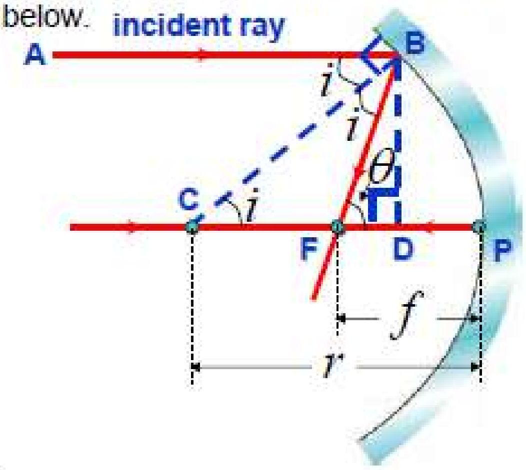
- Consider the ray diagram for concave and convex mirror as shown in figures below.



- From the figures,
  - Point F represents the focal point or focus of the mirrors.
  - Distance *f* represents the focal length of the mirrors.
  - The parallel incident rays represent the object infinitely far away from the spherical mirror e.g. the sun.
- **Focal point or focus, F**
  - for concave mirror – is defined as *a point where the incident parallel rays converge after reflection on the mirror.*
    - Its focal point is real (principal).
  - for convex mirror – is defined as *a point where the incident parallel rays seem to diverge from a point behind the mirror after reflection.*
    - Its focal point is virtual.
- **Focal length, *f***
  - Definition – is defined as *the distance between the focal point (focus) F and pole P of the spherical mirror.*
- The **paraxial rays** is defined as *the rays that are near to and almost parallel to the principal axis.*

2.2.2.Relationship Between focal length(f) and radius of curvature (r)

- Consider a ray AB parallel to the principal axis of concave mirror as shown in figure below.



- From the figure,

  △BCD ⟹ $\tan i = \dfrac{BD}{CD} \approx i$

  △BFD ⟹ $\tan \theta = \dfrac{BD}{FD} \approx \theta$

  Taken the angles are << small by considering the ray AB is paraxial ray.

- By using an isosceles triangle CBF, thus the angle θ is given by

$$\theta = 2i$$

then

$$\frac{BD}{FD} = 2\left(\frac{BD}{CD}\right)$$

$$CD = 2FD$$

- Because of AB is paraxial ray, thus point B is too close with pole P then

$$CD \approx CP = r$$

$$FD \approx FP = f$$

- Therefore

$$\boxed{r = 2f}$$

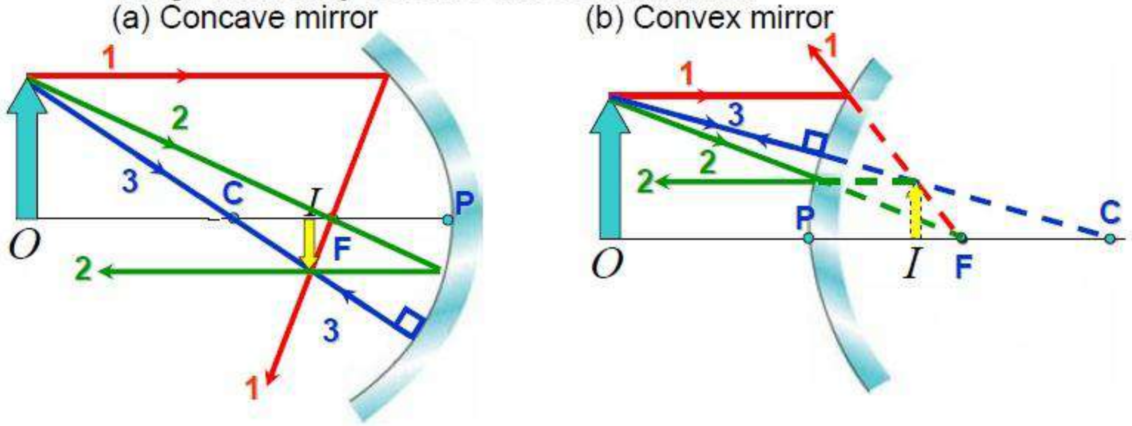
or

$$\boxed{f = \frac{r}{2}}$$

This relationship also valid for convex mirror.

3.2.Ray Diagrams for spherical mirrors

- Definition – is defined as *the simple graphical method to indicate the positions of the object and image in a system of mirrors or lenses.*
- Ray diagrams below showing the graphical method of locating an image formed by concave and convex mirror.

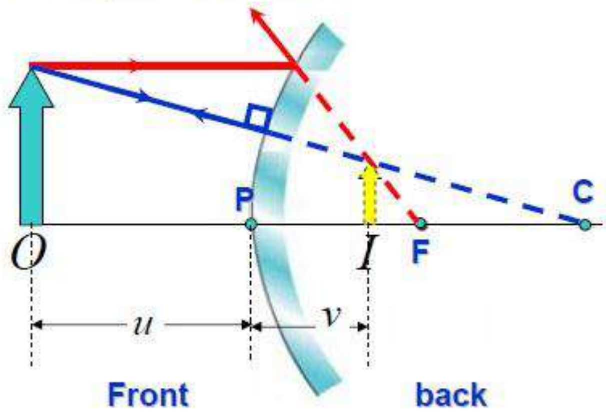


At least any two rays for drawing the ray diagram.

- **Ray 1** - Parallel to principal axis, after reflection, passes through the focal point (focus) F of a concave mirror or appears to come from the focal point F of a convex mirror.
- **Ray 2** - Passes or directed towards focal point F reflected parallel to principal axis.
- **Ray 3** - Passes or directed towards centre of curvature C, reflected back along the same path.

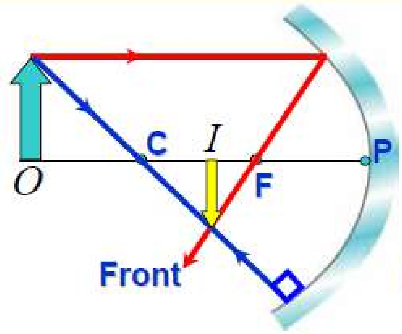
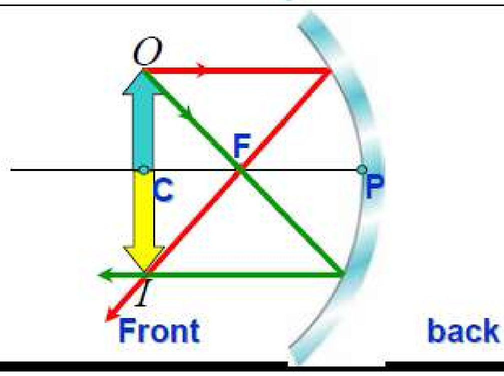
4.2. Images formed by a convex mirrors

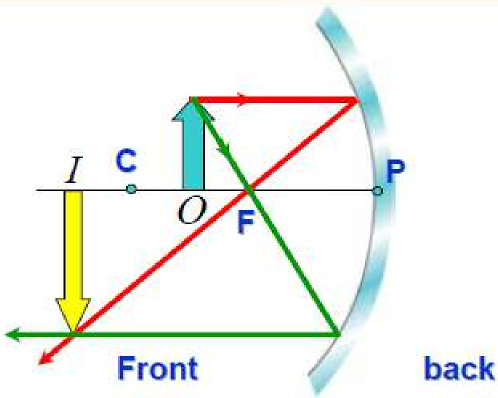
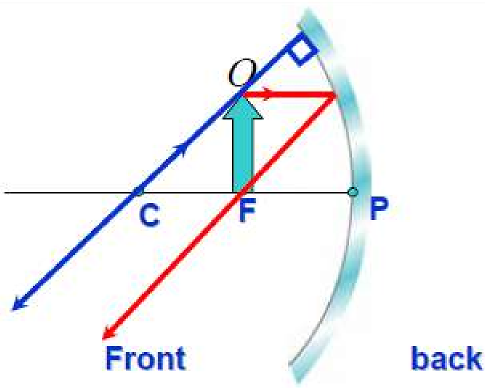
- Ray diagrams below showing the graphical method of locating an image formed by a convex mirror.

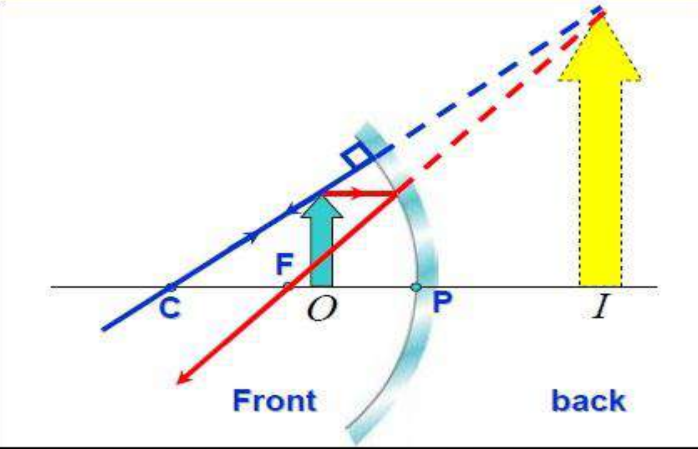


- Properties of image formed are
  - virtual
  - upright
  - diminished (smaller than the object)
  - formed at the back of the mirror
- Object position → any position in front of the convex mirror.

5.2. Images formed by a concave mirrors

Object distance, u	Ray diagram	Image property
$u > r$		○ Real ○ Inverted ○ Diminished ○ Formed between point C and F.
$u = r$		○ Real ○ Inverted ○ Same size ○ Formed at point C.

Object distance, u	Ray diagram	Image property
$f < u < r$		○ Real ○ Inverted ○ Magnified ○ Formed at a distance greater than CP.
$u = f$		○ Real ○ Formed at infinity.

Object distance, u	Ray diagram	Image property
$u < f$		○ Virtual ○ Upright ○ Magnified ○ Formed at the back of the mirror

○ Linear (lateral) magnification of the spherical mirror, M is defined as *the ratio between image height, h_i and object height, h_o*

$$M = \frac{h_i}{h_o} = -\frac{v}{u}$$

where
v : image distance from pole
u : object distance from pole

Negative sign indicates that the object and image are on opposite sides of the principal axis (refer to the real image). If h_o is positive, h_i is negative.

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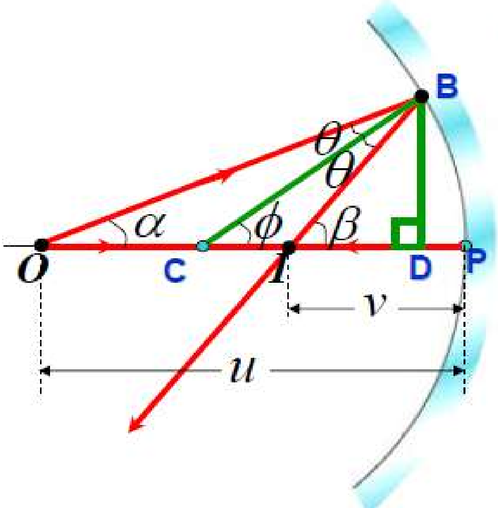
6.2. Derivation of Spherical Mirror Equation

○ Figure below shows an object O at a distance u and on the principal axis of a concave mirror. A ray from the object O is incident at a point B which is close to the pole P of the mirror.



○ From the figure,

$\triangle$BOC $\Longrightarrow$ $\phi = \alpha + \theta$ ……………(1)

$\triangle$BCI $\Longrightarrow$ $\beta = \phi + \theta$ ……………(2)

then, eq. (1)-(2) :

$$\phi - \beta = \alpha - \phi$$

$$\alpha + \beta = 2\phi \quad \text{……………(3)}$$

By using $\triangle$BOD, $\triangle$BCD and $\triangle$BID thus

$$\tan\alpha = \frac{BD}{OD}\,;\ \tan\phi = \frac{BD}{CD}\,;\ \tan\beta = \frac{BD}{ID}$$

○ By considering point B very close to the pole P, hence

$$\tan\alpha \approx \alpha\,;\ \tan\phi \approx \phi\,;\ \tan\beta \approx \beta$$

$$OD \approx OP = u\,;\ CD \approx CP = r\,;\ ID \approx IP = v$$

then

$$\left.\alpha = \frac{BD}{u}\ ;\ \phi = \frac{BD}{r}\ ;\ \beta = \frac{BD}{v}\right\}$$ **Substituting this value in eq. (3)**

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therefore

$$\frac{BD}{u}+\frac{BD}{v}=2\left(\frac{BD}{r}\right)$$

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r} \quad \text{where } r=2f$$

$$\boxed{\frac{1}{f}=\frac{1}{u}+\frac{1}{v}} \Rightarrow \text{Equation (formula) of spherical mirror}$$

- Table below shows the sign convention for equation of spherical mirror .

Physical Quantity	Positive sign (+)	Negative sign (-)
Object distance, u	**Real object** (in front of the mirror)	**Virtual object** (at the back of the mirror)
Image distance, v	**Real image** (same side of the object)	**Virtual image** (opposite side of the object)
Focal length, f	**Concave mirror**	**Convex mirror**
Linear magnification, M	**Upright (erect) image**	**Inverted image**

Example 3:

An object is placed 10 cm in front of a concave mirror whose focal length is 15 cm. Determine

a. the position of the image.

b. the linear magnification and state the properties of the image.

Solution: $u=+10\ cm,\ f=+15\ cm$

a. By applying the equation of spherical mirror, thus

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{15}=\frac{1}{10}+\frac{1}{v}$$

$$v=-30\ cm$$

The image is 30 cm from the mirror on the opposite side of the object (or 30 cm at the back of the mirror).

b. The linear magnification is given by

$$M=-\frac{v}{u}=-\frac{(-30)}{10}$$

$$M=3$$

The properties of the image are

- ✓ **Virtual**
- ✓ **Upright**
- ✓ **Magnified**

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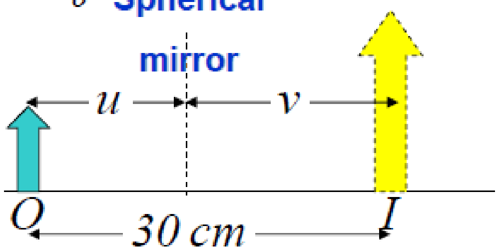
Example 4:

An upright image is formed 30 cm from the real object by using the spherical mirror. The height of image is twice the height of object.

a. Where should the mirror be placed relative to the object?

b. Calculate the radius of curvature of the mirror and describe the type of mirror required.

Solution: $h_i=2h_o$



a. From the figure above,

$$u+|v|=30\ cm \quad \text{(1)}$$

By using the equation of linear magnification, thus

$$M=\frac{h_i}{h_o}=-\frac{v}{u}$$

$$v=-2u \quad \text{(2)}$$

By substituting eq. (2) into eq. (1), hence

$$u=10\ cm$$

The mirror should be placed 10 cm in front of the object.

b. By using the equation of spherical mirror,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{(-2u)}$$

$$f=+20\ cm$$

and $f=\dfrac{r}{2}$ therefore $r=40\ cm$

The type of spherical mirror is **concave** because the positive value of focal length.

Example 5:

A mirror on the passenger side of your car is convex and has a radius of curvature 20.0 cm. Another car is seen in this side mirror and is 11.0 m behind the mirror. If this car is 1.5 m tall, calculate the height of the car image . (Similar to No. 34.66, pg. 1333, University Physics with Modern Physics,11th edition, Young & Freedman.)

<u>Solution</u>: $h_o=1.5x10^2\ cm,\ r=-20.0\ cm,\ u=+11.0x10^2\ cm$

By applying the equation of spherical mirror,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v} \quad \text{and} \quad f=\frac{r}{2}$$

$$\frac{2}{r}=\frac{1}{u}+\frac{1}{v}$$

$$v=-9.91\ cm$$

From equation of linear magnification,

$$M=\frac{h_i}{h_o}=-\frac{v}{u}$$

$$h_i=-\left(\frac{v}{u}\right)h_o$$

$$h_i=1.35\ cm$$

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Example 6: (H.W.)

a. A concave mirror forms an inverted image four times larger than the object. Find the focal length of the mirror, assuming the distance between object and image is 0.600 m.

b. A convex mirror forms a virtual image half the size of the object. Assuming the distance between image and object is 20.0 cm, determine the radius of curvature of the mirror.

Ans. : 160 mm, -267 mm