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3-

Electromagnetic energy

The electrostatic potential energy of the system is:

$$w = \frac{1}{2}\int_V \bar{E}\cdot\bar{D}\, dv \quad (1)$$

Similirty the energy Store in the magnetic field

$$w_m = \frac{1}{2}\int_V \bar{B}\cdot\bar{H}\, dv \quad (2)$$

Using maxwell equation to ~~der~~ drive the two equation

$$\boxed{\mathrm{div}(\bar{E}\times\bar{H}) = -\frac{\partial}{\partial t}\frac{1}{2}(\bar{E}\cdot\bar{D}+\bar{B}\cdot\bar{H}) - \bar{J}\cdot\bar{E}} \quad (3)$$

$$\boxed{-\int_V \bar{J}\cdot\bar{E}\, dv = \frac{d}{dt}\int \frac{1}{2}(\bar{E}\cdot\bar{D}+B\cdot H)\, dv + \oint_S \bar{E}\times\bar{H}\cdot\hat{n}\, ds} \quad (4)$$

Prove that:

The solution of eqn-(3) by using Maxwell eqn-

$$\bar{H}\cdot\mathrm{curl}\,\bar{E} - \bar{E}\cdot\mathrm{curl}\,\bar{H} = H\cdot\nabla\times\bar{E} - \bar{E}\cdot\nabla\times\bar{H} \quad (5)$$

$$= -\bar{H}\cdot\frac{\partial \bar{B}}{\partial t} - \bar{E}\left(\bar{J}+\frac{\partial \bar{D}}{\partial t}\right)$$

$$= -H\cdot\frac{\partial \bar{B}}{\partial t} - \bar{J}\cdot\bar{E} + E\cdot\frac{\partial \bar{D}}{\partial t} \quad (6)$$

But $\operatorname{div}(\bar{A}\times\bar{B}) = \bar{B}\cdot\operatorname{curl}\bar{A} - A\cdot\operatorname{curl}\bar{B}$

Therefor: $\operatorname{div}(E\times H) = H\cdot\operatorname{curl}\bar{E} - \bar{E}\cdot\operatorname{curl}\bar{H}$ ---- (7)

substituting eqn. (7) in equation (6) we obtain:

$$\operatorname{div}(\bar{E}\times\bar{H}) = -\bar{H}\cdot\frac{\partial \bar{B}}{\partial t} - \bar{J}\cdot\bar{E} - \bar{E}\cdot\frac{\partial \bar{D}}{\partial t} \quad \text{—— (8)}$$

For eqn-(8):

a) $\bar{H}\cdot\frac{\partial B}{\partial t} = H\cdot\frac{\partial}{\partial t}\mu H = \frac{1}{2}\mu\frac{\partial}{\partial t}H^2 = \frac{1}{2}\frac{\partial}{\partial t}\bar{H}\cdot\bar{B}$ --- (8-a)

thus: $\bar{B} = \mu\bar{H}$

b) $\bar{E}\cdot\frac{\partial \bar{D}}{\partial t} = \bar{E}\cdot\frac{\partial}{\partial t}\epsilon\bar{E} = \frac{1}{2}\epsilon\frac{\partial}{\partial t}E^2 = \frac{1}{2}\frac{\partial}{\partial t}\bar{E}\cdot\bar{D}$ --(8-b)

Thus: $\bar{D} = \epsilon\bar{E}$

substitute: (8-a) and (8-b) in equation (8) we obtain:

$$\operatorname{div}(\bar{E}\times\bar{H}) = \frac{1}{2}\frac{\partial}{\partial t}\bar{H}\cdot\bar{B} - \frac{1}{2}\frac{\partial}{\partial t}\bar{E}\cdot\bar{D} - \bar{J}\cdot\bar{E}$$

$$\boxed{\operatorname{div}(\bar{E}\times\bar{H}) = -\frac{1}{2}\frac{\partial}{\partial t}(\bar{E}\cdot\bar{D} + \bar{H}\cdot\bar{B}) - \bar{J}\cdot\bar{E}} \quad \text{— (9)}$$

Equation (9) is the same eqn. (3). So that eqn. (9) is ~~shoudthat~~ showing eqn. (3).

ان معادلة (9) هي نفسها معادلة (3) اي ان معادلة (9) تعني اثبات معادلة (3) أو اشتقاقها.