Analytical Machanics

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Level (3)

Chapter (1)

1-1 Fundamental Concepts Vectors :-

The Vector $\bar{A} = A_x + jA_y + KA_z$

means That there vector (A) is expressed on the right in terms of its components in a particular Coordinate system.

1-2 Foemal Definition and Rules.

1. Equality of vectors.
   $\vec{A} = \vec{B}$
   $[A_x, A_y, A_z] = [B_x, B_y, B_z]$
   $A_x = B_x, A_y = B_y, A_z = B_z$
   Two vectors are equal if, and if their respective components are equal.
2. Vector Addition
   A+B $\Longrightarrow [A_x + B_x, A_y + B_y, A_Z + B_z]$
3. Multiplication by a scalar
   if c is a scalar and A vector
   cA = $[cA_x, cA_y, cA_z]$
4. Vector subtraction
   A-B = $A_x(-B) = [A_x - B_x, A_y - B_y, A_z - B_z]$
5. The null vector .
   O = [o, o, o]
6. The combative of Low of Addition
   A+B = B+A
   $A_x + B_x = B_x + A_x$, and similarly for the and y Components.
7. The Associative Law.
   A_x + (B+C) = $[A_x + (B_x + C_x), A_y + \left(B_y + C_y\right), A_z + (B_z + C_z)]$
   = $[\ (A_x + B_x) + C_x, \left(A_y + B_y\right) + C_y, (A_z + B_z) + C_z)]$
   = $(A + B) + C$
8. The Distributive Law
   c [A+B] = c $[A_y + B_x, A_y + B_y, A_z + B_z]$
   = $[cA_x + cB_x, cA_y + cB_y, cA_z + cB_z]$
   = $cA + cB$

<u>1-3</u> Magnitude of avector

$$A = |A| = \left(A_x^2 + A_y^2 + A_y^2\right]^{\frac{1}{2}}$$

1-4 Unite Coordinate vector

$i = [\ 1\ ,0\ ,0\ ],\ \ j = [0\ ,1\ ,0\ ] \quad , \quad k = [\ 0\ ,0\ ,1]$

$A = iA_x + jA_y + kA_z$

<u>1–5</u> The scalar Product

$* \ A.B = A_x B_x \ + \ \ A_y B_y + A_z B_z$

$* \ A.B = B.A$

$* \ A.B = AB \cos\theta$

$* \ \cos\theta = \frac{A.B}{AB}$

$* \ A.(B + C) = A.B + A\ .C$

From the definuts of the unite Cordinnate Vectors i , j and k , it is clear that the following relations hold

$i\ .i = j = j = k\ .k = 1$

$i\ .j = i\ .k = j\ .k = 0$

$*$ The Condition for static equilibrium of the particle the Particle willnot Mave underthe action of those forcc $F_1 + F_2 + \cdots + F_n => \sum F_i = 0$

1–6 the vector Product

$* \ \bar{A} \times \bar{B} = [(A_y B_z - A_z B_y), (A_z B_x - A_x B_z), (A_x B_y - A_y B_x)].$

$[(A_x B_y - A_y B_x)].$

$* \ A \times B = Ab \sin\theta$

$* \ \bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$

$* \ A \times (B + c) = (A \times B) + (A \times c)$

$* \ i \times i = j \times j = k \times k = 0$

$* \ j \times k = i = -k \times j$

EX :

prove That $i \times j = k$

$i \times j = (0 - 0, 0 - 0, 1 - 0) = (0, 0, 1) = k$

$|\vec{A} \times \vec{B}|^2 = (A_y B_z - A_z B_y)^2 + (A_z B_x - A_x B_z)^2 + (A_x B_y - A_y B_x)^2$

$|\vec{A} \times \vec{B}|^2 = \overrightarrow{A^2}\,\overrightarrow{B^2} - (\bar{A}.\bar{B})^2$

example :- given the two vectors

$\vec{A} = 2i + j - k, \vec{B} = i - j + 2k, find$

1. $A.B$
2. $A \times B$
3. unit Vector normal to the plane the two vector .
4. The ungle between A and B .

Sol :-

1- A.B = 2 – 1 – 2 = - 1

2- $A \times B = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = i(2-1) + j(1-4) + k(-2-1)$

$= i - 3j - 3k$

$$\cos\theta = \frac{A.B}{Ab} = \frac{-1}{(2^2+1^1+(-1)^2)^{\frac{1}{2}}+(1^2+(-1)^2+2^2)^{\frac{1}{2}}}$$

$$= \frac{-1}{6} => \quad \emptyset = 99.6^o$$

3- $n = \frac{A\times B}{|A\times B|} = \frac{i-3j-3k}{(1)^2+(-1)^2+(-3)^2]^{\frac{1}{2}}}$

$$= \frac{i-3j-3k}{(1-9+9)^{\frac{1}{2}}} = \frac{i}{\sqrt{19}} - \frac{3}{\sqrt{19}}\, j - \frac{3}{\sqrt{19}} k$$

1–7 Moment of a force

* The moment of aforce about appoint is a vector quantity having a magnitude and direction

$$\vec{N} = \vec{r} \times \vec{F}$$

$$|\vec{N}| = |\vec{r} \times \vec{F}| = r\, F \sin\theta$$

$*$ The Condition for rotational equilibrium is that the vector sum of all the moments is zero>

i.e $\sum \vec{r_i} \times \vec{F_i} = \sum N_i = 0$

1–8 change of coordinate system :-

$\vec{A} = \vec{\imath}A_x + \vec{\jmath}A_y + \vec{K}A_z$

$\vec{A}' = i'A'_x + j'A'_y + k'A_y.$

$A'x = \vec{A}.i' = (i.i')A_x + (j.j')A_y + (k.k')A_z$

$A'y = \vec{A}.j' = (i.j')A_x + (j.j')A_y + (k.j')A_z$

$A'z = \vec{A}.i' = (i.k')A_x + (j.k')A_y + (k.k')A_z$

$$\vec{A} = \begin{vmatrix} i,i' & j,j' & k.i' \\ i.j' & j,j' & k,j' \\ i.k' & j.k' & k.k' \end{vmatrix} \Longrightarrow \text{coefficientd of transformation}$$

= they are equal to the cosines of the axes of the primed coordinate system

$$\begin{bmatrix} A_x - \\ A_y - \\ A_z - \end{bmatrix} = \begin{bmatrix} i,i' & j,j' & k.i' \\ i.j' & j,j' & k,j' \\ i.k' & j.k' & k.k' \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Transformation Matrix

example :-

Express the vector $\vec{A} = 3i + 2j + k$ in terms of thetrid $i'j'k'$ where the $x'y'$ axes rotated 45° around the z axis.

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$A'_x = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

$A'_y = \frac{-3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$A'_z = 0 + 0 + 1 = 1$

$\therefore A' = \frac{5}{2} i' - \frac{1}{\sqrt{2}} j' + k'$

H.W

$Q.(a, c) - Q_2(b) - Q_3, Qu, Q_5, Q_6, Q_7, Q_{13}$