- By using the equation of spherical refracting surface, the refraction by first surface AB and second surface DE are given by
  - **Convex surface AB ($r = +r_1$)**

$$\frac{n_1}{u_1} + \frac{n_2}{v_1} = \frac{(n_2 - n_1)}{r_1} \quad \text{------------(1)}$$

  - **Concave surface DE ($r = -r_2$)**

$$\frac{n_2}{(t - v_1)} + \frac{n_1}{v_2} = \frac{(n_1 - n_2)}{-r_2}$$

Assuming the lens is very thin thus $t = 0$,

$$\frac{n_2}{-v_1} + \frac{n_1}{v_2} = \frac{(n_1 - n_2)}{-r_2}$$

$$\frac{n_2}{v_1} = -\left[\left(\frac{n_1 - n_2}{-r_2}\right) - \frac{n_1}{v_2}\right]$$

$$\frac{n_2}{v_1} = \frac{n_1}{v_2} - \left(\frac{n_2 - n_1}{r_2}\right) \quad \text{------------(2)}$$

- By substituting eq. (2) into eq. (1), thus

$$\frac{n_1}{u_1} + \left[\frac{n_1}{v_2} - \left(\frac{n_2 - n_1}{r_2}\right)\right] = \frac{(n_2 - n_1)}{r_1}$$

$$\frac{n_1}{u_1} + \frac{n_1}{v_2} = \frac{(n_2 - n_1)}{r_1} + \frac{(n_2 - n_1)}{r_2}$$

then

$$\frac{1}{u_1} + \frac{1}{v_2} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \text{------------(3)}$$

- If $u_1 = \infty$ and $v_2 = f$ hence eq. (3) becomes

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Lens maker's equation

where

f : focal length
r_1 : radius of curvature of first refracting surface
r_2 : radius of curvature of second refracting surface
n_1 : refractive index of the medium
n_2 : refractive index of the lens material

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- By equating eq. (3) with lens maker's equation, hence

$$\frac{1}{u_1}+\frac{1}{v_2}=\frac{1}{f}$$

therefore in general,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$ → **Thin lens formula**

- Note :
  - If the medium is **air** ($n_1 = n_{air} = 1$) thus the lens maker's equation will be

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

where n : refractive index of the lens material

  - For thin lens formula and lens maker's equation, Use the **sign convention** for **refraction**. → **Very Important**
  - The radius of curvature of flat refracting surface is infinity, $r = \infty$.

Example 8 :

A biconvex lens is made of glass with refractive index 1.52 having the radii of curvature of 20 cm respectively. Calculate the focal length of the lens in

a. water,

b. carbon disulfide.

(Given n_w = 1.33 and n_c=1.63)

Solution: $r_1=+20\ cm,\ r_2=+20\ cm,\ n_g=n_2=1.52$

a. Given the refractive index of water, $n_w = n_1$

By using the lens maker's equation, thus

$$\frac{1}{f}=\left(\frac{n_g}{n_w}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$f=+70\ cm$$

b. Given the refractive index of carbon disulfide, $n_c = n_1$

By using the lens maker's equation, thus

$$\frac{1}{f}=\left(\frac{n_g}{n_c}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$f=-148.18\ cm$$

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Example 9 :

A converging lens with a focal length of 90.0 cm forms an image of a 3.20 cm tall real object that is to the left of the lens. The image is 4.50 cm tall and inverted. Find

a. the object position from the lens.

b. the image position from the lens. Is the image real or virtual?

No. 34.26, pg. 1331, University Physics with Modern Physics,11th edition, Young & Freedman.

Solution: $f=+90.0\ cm,\ h_o=3.20\ cm,\ h_i=-4.50\ cm$

a. By using the linear magnification equation, hence

$$M=\frac{h_i}{h_o}=-\frac{v}{u}$$

$$v=1.41u \quad \text{------------(1)}$$

By applying the thin lens formula,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{90.0}=\frac{1}{u}+\frac{1}{v} \quad \text{------------(2)}$$

By substituting eq. (1) into eq. (2),hence

$$u=154\ cm$$

The object is placed 154 cm in front of the lens.

b. By substituting $u=154\ cm$ into eq. (1),therefore

$$v=217\ cm$$

The image forms 217 cm at the back of the lens (at the opposite side of the object placed) and the image is real.

Example 10 :

An object is placed 90.0 cm from a glass lens (n=1.56) with one concave surface of radius 22.0 cm and one convex surface of radius 18.5 cm. Determine

a. the image position.

b. the linear magnification. (Gc.862.28)

Solution: $u=+90.0\ cm,\ n=1.56,\ r_1=-22.0\ cm,\ r_2=+18.5\ cm$

a. By applying the lens maker's equation in air,

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$f=+208\ cm$$

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By applying the thin lens formula, thus

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$v=-159\ cm$$

The image forms 159 cm in front of the lens (at the same side of the object placed)

b. By applying equation of linear magnification for thin lens, thus

$$M=-\frac{v}{u} \Longrightarrow M=1.77$$

Example (11): H.W

A glass (n=1.50) plano-concave lens has a focal length of 21.5 cm. Calculate the radius of the concave surface. (Gc.862.26)

Ans. : -10.8 cm

Example (12): H.W

An object is 16.0 cm to the left of a lens. The lens forms an image 36.0 cm to the right of the lens.

a. Calculate the focal length of the lens and state the type of the lens.

b. If the object is 8.00 mm tall, find the height of the image.

c. Sketch the ray diagram for the case above. (UP. 1332.34.34)

Ans. : +11.1 cm, -1.8 cm 58