Chapter five

Numerical Differentiation and Integration

(1)If $x_{i+1} - x_i = h$

A. Numerical Differentiation of Newton Forward

$$f(x) = f(x_0) + (x - x_0)\frac{\Delta f_0}{1!\,h} + (x - x_0)(x - x_1)\frac{\Delta^2 f_0}{2!\,h^2} + \cdots$$
$$+ (x - x_0)(x - x_1)\ldots(x - x_n)\frac{\Delta^n f_0}{n!\,h^n} \ldots\ldots\ldots\ldots (1)$$

Example: find $f'(2.5)$, $f''(2.5)$, *and* $f'(3)$ *from the table :*

x	2	3	4	5	6
f(x)	5	10	17	26	37

Solution : we used Newton Forward Different formula (chapter four) to get the following table:

X	f	Δf_0	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_0$
2	5				
3	10	5	2	0	
4	17	7	2	0	0
5	26	9	2		
6	37	11			

$$q=\frac{x-x_0}{h}=\frac{2.5-2}{1}=0.5$$

$$f'(x)=\frac{1}{h}\Big[\Delta f_0+\frac{1}{2}(2q-1)\Delta^2 f_0+\frac{1}{6}(3q^2-6q+2)\Delta^3 f_0 + \frac{1}{24}(4q^3-18q^2-22q-6)\Delta^4 f_0+\cdots\Big]$$

$$f'(2.5)=\frac{1}{1}\Big[5+\frac{1}{2}\big(2(0.5-1)\big)*2+\frac{1}{6}(3(0.5)^2-6(0.5)+2*0+0)\Big]$$

$$f'(2.5)=5$$

$$f''(x)=\frac{1}{h^2}\Big[\Delta^2 f_0+(q-1)\Delta^3 f_0+\frac{1}{12}(6q^2-18q+11)\Delta^4 f_0+\cdots\Big]$$

$$f''(2.5)=\frac{1}{(1)^2}[2+0+0]=2$$

$$f'(x_0)=\frac{1}{h}\Big[\Delta f_0-\frac{1}{2}\Delta^2 f_0+\frac{1}{3}\Delta^3 f_0+\frac{1}{4}\Delta^4 f_0+\cdots\Big]$$

$$f'(3)=\frac{1}{1}\Big[7-\frac{1}{2}*2+\frac{1}{3}*0+\frac{1}{4}*0\Big]=6$$