Chapter six

Numerical Integration

To evaluate $\int_a^b f(x)dx$, we divide the interval [a ,b] in to n subintervals, which have same length i.e $h = \frac{b-a}{n}$.

$$x_0 = a\, , x_n = b\, , x_i = x_0 + ih\, , \quad i = 1,2,3, \dots , n-1$$

Let $f(x) \cong P_n(x)$

$$\therefore \int_a^b f(x)dx = \int_a^b P_n(x)dx \qquad (1)$$

We approximate $P_n(x)$ by Newton forward difference interpolating

$$P_n(x) = f_0 + (x-x_0)\frac{\Delta f_0}{1!\,h} + (x-x_0)(x-x_1)\frac{\Delta^2 f_0}{2!\,h^2} + \cdots + (x-x_0)(x-x_1)\dots(x-x_n)\frac{\Delta^n f_0}{n!\,h^n}$$

$$\therefore \int_a^b f(x)dx = \int_{x_0}^{x_n} (f_0 + (x-x_0)\frac{\Delta f_0}{1!\,h} + (x-x_0)(x-x_1)\frac{\Delta^2 f_0}{2!\,h^2} + \cdots + (x-x_0)(x-x_1)\dots(x-x_n)\frac{\Delta^n f_0}{n!\,h^n})dx. \qquad (2)$$

Let $\frac{x-x_0}{n} = \text{q} \rightarrow x - x_0 = \text{hq}.$

$\rightarrow x - x_1 = \text{h}(q-1)$

$x - x_2 = \text{h}(q-2)$

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$x - x_n = \text{h}(q-n) \;\; and \;\; dx = hdq$

If $x = x_0 \rightarrow q = 0$ $if\ x = x_n \rightarrow q = n.$

Substitute the last relation in to (2) we have

$$\therefore \int_a^b f(x)dx = \int_0^n (f_0 + q\Delta f_0 + \frac{q(q-1)}{2}\Delta^2 f_0 + \frac{q(q-1)(q-2)}{6}\Delta^3 f_0 + \cdots)hdq.$$

$$\therefore \int_a^b f(x)dx = h\int_0^n (f_0 + q\Delta f_0 + \frac{q(q-1)}{2}\Delta^2 f_0 + \frac{q(q-1)(q-2)}{6}\Delta^3 f_0 + \cdots)dq \quad \ldots (3)$$

The equation (3) is called Newton – cost formula .

The homework is use Newton Backword – difference interpolating formula to obtain Newton – cost formula .