Chapter six

Numerical Integration

(1)Trapezoidal Formula:

Let n= 1 , [a,b] = $[x_0, x_1]$ *from (3) we have :*

$$\therefore \int_a^b f(x)dx = h\int_0^n \left(f_0 + q\Delta f_0 + \frac{q(q-1)}{2}\Delta^2 f_0 + \frac{q(q-1)(q-2)}{6}\Delta^3 f_0 + \cdots\right)dq$$

$$\int_a^b f(x)dx = h\left[f_0 q + \frac{q^2}{2}\Delta f_0 + \frac{1}{2}\left(\frac{q^3}{3} - \frac{q^2}{2}\right)\Delta^2 f_0 + \cdots\right]_0^1$$

$$= h\left(f_0 + \frac{1}{2}\Delta f_0 - \frac{1}{12}\Delta^2 f_0 + \ \ldots\ \right)$$

$$= h\left(f_0 + \frac{1}{2}(f_1 - f_0) - \frac{h}{12}\Delta^2 f_0\right) \qquad (\Delta f_0 = f_1 - f_0)$$

$$\therefore \int_a^b f(x)dx = \frac{h}{2}(f_0 + f_1) - \frac{h}{12}\,\Delta^2 f_0$$

$$\int_a^b f(x)dx \cong \frac{h}{2}(f_0 + f_1) \quad \ldots \quad (4)$$

$$\therefore E = \frac{-h}{12}\,\Delta^2 f_0$$

$$f''(\zeta) = \frac{1}{h^2}(\Delta^2 f_0 + (q-1)\Delta^3 f_0 + \cdots)$$

$$\therefore f''(\zeta) = \frac{1}{h^2}\Delta^2 f_0 \Rightarrow E = \frac{-h^3}{12}f''(\zeta) \quad \text{where } \zeta \in [x_0, x_n] \quad \ldots (5)$$

If $[a,b] = [x_0, x_n]$

$\therefore\ [x_0, x_n] = [x_0, x_1],\ [x_1, x_2], \ldots,\ [x_{n-1}, x_n]$

$$\int_a^b f(x)dx = \int_{x_0}^{x_n} f(x)dx$$

$$= \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \ldots + \int_{x_{n-1}}^{x_n} f(x)dx$$

From (4) we have :

$$\int_a^b f(x)dx = \frac{h}{2}(f_0 + f_1) + \frac{h}{2}(f_1 + f_2) + \cdots + \frac{h}{2}(f_{n-1} + f_n)$$

$$\int_a^b f(x)dx = \frac{h}{2}\,[f_0 + 2f_1 + 2f_2 + \cdots + f_n] \quad \ldots (6)$$

From (5) we have :

$$E_T = \frac{-nh^3}{12} f''(\zeta) \quad \text{where } \zeta \in [x_0, x_n] \cong [a, b] \ \ldots (7)$$

The equation (6) is called Trapezoidal formula and (7) is called the error of the method .