Chapter two

:Definition :1_2

. Let S be a non-empty subset of aring R

, Then (S , + , .) is a subring of R if a – b ∈ S And a

b ∈ S And we can define a subring as we see in the
:following definition

. Definition: Let (R , + , .) be a ring and let ∅ ≠ ⊊ R

Then (S , + , .) is a subring if and only if

a – b ∈ S , ∀ a , b ∈ S ①

a . b ∈ S , ∀ a , b ∈ S Remark: Each ring has at ②
. lest two subrings are { 0 } and a ring itself

Called trivial subrings

Examples .

is a subring of a ring (Z , + , .) (. , + Ze)❶

is subring of ($Z6$,+6 , . (6. , 6+ , { 4 , 2 , 0})❷
6)

subring of (R , + , .) where S = { a + (. , + , S)❸
√3b : a , b ∈ Z }

Solution

3 √ (b – d) + (a – c) = (c + d√3) - (a + b√3)①
 ∈ S

ad +) + (ac + 3bd) = (c + d√3) . (a + b√3)②
3 ∈ S√ (bc

is a Subring from (Z , + , .) But (. , + Ze)③
(Z0+ , .) is not subring of (Z , + , .) [because 7 - 5
 = 2 ∉ Z0]

:Remarks.2-5

Let (R , + , .) be a ring and let (S , + , .) be a
. subring of R

 Then 1 اذا كانت R تمتلك محايد ضربي . ليس
شرطا ان تكون لها محايد 2 في بعض االحيان الحلقة
 الجزئية لها محايد بينما الحلقة R ليس لها محايد
 ضربي . 3احيان آخرى x تمتلك محايد ضربي. بينما s
. تمتلك محايد ضربي مختلف

 :Examples

Let (R X R , + , .) be a ring and (R X 0 , + , .) is a
, Subring of R

. define + , ∙ as the Following

and (a , b) . (c , d) (a + c , b + d) = (c , d) + (a , b)
∋ = (ac , bd) ∀ (a , b) , (c , d)

Then (1 , 1) ∈ R X R is the identity element of ring

But (1 , 0) ∈ R X 0 is the identity element, of
. subring

:Remarks 2-6

. Let (R , + , .) be a ring and Let a ,b ∈ R. n , m ∈ Z

Then

a = na + ma. ex (2 + 3) a = 2a + 3a (n + m)①

a = n (ma). ex (2 . 3) a = 2 (3a) (nm)②

n (a + b) = na + nb. ex 4(a + b) = 4a + 4b 14 ③

n. (a . b) = (na) . b = a. (nb). ex 5(a . b) = (5a)b = ④
a (5b)

ex (5a) . (2b) = (5.2). .(a . b) (nm) = (mb) .(na)⑤
(a . b)

:Definition : 7_2

Let (R , + , .) be a ring. Then the set Z(R) = {a ∈ R : x , a = a . x ,∀x ∈ R} is called the center of the ring . R