Chapter One

نظرية الحلقات

Theory Ring

Definition: By a ring we mean a non- 1_1 empty set R with two binary operations * and ° is called a ring and is denoted by (R , * , °) if

is a belian group (+ ,R) -1

.is a semi-group (° ,R) -2

. * is distributive (on both sides) over ° -3

Another defini tion : it A mathematical system (R , * , °) is called a ring if

. a , b ∈ R , a ∗ b ∈ R (closed) under ∀ -1

.a ∗ b = b ∗ a ∀ a , b ∈ R (commutative) -2

c = a ∗ (b ∗ c) ∀ a , b ,c ∈ R ∗ (a ∗ b) -3
. (associative)

e ∈ R ∋ a ∗ e = e ∗ a = e ∀ a , b ,c ∈ R ∃ -4
. (identity)

$a \in R \ \exists \ -a \in R \ni a \ast (-a) = (-a) = e \ \ \forall$ -5
. (inverse)

. $a , b \in R , a \circ b \in R$ (closed)under $\forall$ -6

$c = a \circ (b \circ c) \ \forall \ a , b , c \in R \circ (a \circ b)$ -7
. (associative)

$a \circ (b \ast c) = (a \circ b) \ast (a \circ c)$ (two sides -8
distributive) and $(b \ast c) \circ a = (b \circ a) \ast (c \circ a$
$)$ ($\forall \ a , b, c \in R$)

. Some Important examples of rings : 2 – 1

is a ring , such that R is the real (·, + ,R) -1
number

if $R =\{ a + b\sqrt{3} : a, b \in Z \}$, then (R, + , •) is a -2
. ring

is a ring, where P(X) is a Power ($\cap$, Δ, P(X)) -3 3
,set of a set X

and Δ is a symmetric difference $A \Delta B = (A - B)$
. $\cup(B - A)$

is a ring . (P(X) , Δ ,$\cap$) is a ring : But ($\cap$, Δ , P(X))
. (P(X) , Δ , $\cup$) is not ring

Now, to prove the intersection is distribute to a .seem ethic difference

First, we need the following. A ∩ (B − C) = (A ∩ B .) −(A ∩ C)

It's clear that (P (X) , Δ) is a belian group, and (P .(X) , ∩) is a semi-group

A ∩ (B Δ C) = A ∩ [(B – C) U (C – B)] = [A ∩ (B – C)] U [A ∩ (C – B)] = [(A ∩ B) − (A ∩ C)] U .[(A ∩ C)−(A ∩ B)] = (A ∩ B) Δ (A ∩ C)

Note : A U (B – C) ≠ (A U B) – (A U C)