:Definitions 1-10

An element x in a ring R is called a right zero divisor it there exists a non-zero element $y \in R$ Such that y. x = 0

Also , we can similarly define left zero divisor if x . . y = 0

An element is called a zero divisor if it is both right .and left zero divisor

:Definition : 11 -1

A ring (R , + , •) called divisors of Zero if there ,exists tow non-Zero elements a , b ∈ R

That a . b = 0

Examples: (Z4 ,+4 , .4) 2 ∈ Z4 and 2 .4 2 = 0 (Z6 ,+6 , .6) , 2 , 3 ∈ Z6

0 = 3 . 2 $\Longrightarrow$

:Proposition

A ring (R , + , •) is without zero divisors if and . only if the canselation low hold

:Proof 11

Let (R , + , •) is a ring without zero divisors. and , let a , b , c ∈ R

such that a ≠ 0 and a . b = a . c ⟹ a . b − a − c = 0 ⟹ a . (b - c) = 0

b - c = 0 (because a ≠ 0) . Thus b = c ⟹

;Conversely

let a ,b ∈ R and a ≠ 0 to , such that a . b = 0

a . b = a . 0 ⟹ b = 0 (By canselation low) ⟹

And by same way we can show that if b ≠ 0 ⟹ . we get a = 0

. Thus (R , + , •) is aring without zero divisors

Corollary .1.13

let (R , + , •) be a ring with identity and without .zero divisors

If $a^2 = a$, where a ∈ R , then either a = 0 or a = 1

:Proof

Suppose a ≠ 0 $a^2 = a \Longrightarrow a^2 - a = 0 \Longrightarrow a (a - 1) = 0$

. Since a ≠ 0 , then a -1 = 0

. Hence a = 1 And if a ≠ 1 , then a – 1 ≠ 0

. Thus a = 0

:Definition :1-14

A ring (R , * , °) is called an integral domain if R is a commutative ring with identity and without zero .divisors

Definition: A commutative ring with :1.15 . identity

is called a field if each now zero (. , + , F) element has an inverse under multiplication . (invertible)

Note : each field has no zero divisors 1-16:
. Examples (Q*, + , .) and (R*, + , .) are Fields