

, But $(R, +, \circ)$ is not a ring

because the left distributive law is not achieved $f \circ$

$$\cdot (g + h) \neq (f \circ g) + (f \circ h)$$

. But $(\text{hom } R, +, \circ)$ is a ring with identity

$$\cdot \text{Where } (f + g)(a) = f(a) * g(a)$$

solution: The Proof of $(\text{hom } G, +)$ is a belian group

is simmilarly to the previous proof

Now, to Prove $(\text{hom } G, \circ)$ is semi group

$$f(a) \circ g(a) \in R \quad (f \circ g) \circ h)(a) = (f \circ g)(a) \circ h(a)$$

$$(a) \circ h(a) = (f(a) \circ g(a)) \circ h(a)$$

$$f(a) \circ (g(a) \circ h(a)) = f(a) \circ ((g \circ h)(a)) = (f \circ (g \circ h))(a)$$

. Hence $(\text{hom } G, \circ)$ is semi group

Note: The inverse of each function in $\text{hom } G$ is $\forall f \in \text{hom } G \exists -f \in \text{hom } G$

such that $(-f)(a) = f(a)^{-1} \in \text{hom } G$ And the closed

,

we can show that $(f + g)(a * b) = f(a * b) * g(a * b) = f(a) * f(b) * g(a) * g(b) = f(a) * g(a) * f(b) * g(b) = (f + g)(b)$

: Finally ; we must prove the distributive law

$f((g + h)(a)) = f(g(a) + h(a)) = (a)[f \circ (g + h)]$
 $f(g(a)) * f(h(a)) = (f \circ g)(a) * (f \circ h)(a) = (f \circ g + f \circ h)(a)$

,By the Same way

we can prove that $(f \circ (g + h))(a) = (f \circ g + f \circ h)(a)$

. Therefore $(\text{hom } G, +, \circ)$ is a ring