:Remark :8–2

.The center of a ring is a subring

:Proof

.∅ ≠ Since 0 . x = x .0 ⟹ 0 ∈ Z (R) ⟹ Z(R)

Let a – b ∈ Z(R) , then (a - b) x = ax – bx = xa – xb = x(a - b)

Thus a – b ∈ Z(R) Furthers , (a . b)x = a(bx) = a(xb) = (ax) b = (xa) b = x(ab)

Hence a . b ∈ Z(R) Therfore Z(R) is a subring of a . ring R

.Remark: [f R is a com . ring . Then cent . R = R

:Definition 2-9

Let S be a subset of a ring R. Then the -Smallest subring of R containing S Is called the subring .generated by S

:Definition 2-10

If there exists a positive integer n , Such that na = .0 for each clement a ∈ R

The smallest such Positive integer is called the .characteristic of R

If no such positive integer exists , R is said to lave .characteristic zero

.The characteristic of R is denoted char R

:Examples:2-11

.is a ring has characteristic zero (. , + , Z)

And (n ,+n , .n) is a ring of integers modulen has . characteristic n

Also (Q , + , .) and (R , + , .) are rings have .characteristic Zero

.is a ring has characteristic 2 (∩ , Δ , P(x))

Because ∀ A ∈ P(X). 2A = A Δ A = (A - A) U (A - A) . ∅ = ∅ U

:Theorem :2-12

,Let (R , + , .) be a ring with identity

then (R , + , .) has a characteristic n > 0 if and only if n is the lest positive integer such that n . 1 = 0

:Proof

. If n is not a lest positive integer such that n. 1 = 0

Thus there exists m such that n > m > 0 and m .1 = . 0

. Then , we get m . a = m . (1 . a) = (m . 1). a = 0

a = 0 ∀ ∈ R it's mean that the charactristic of R is , leas than n

which is a contradiction . Thus n is the lest Positive integer such that n.1 =0

;Conversely

, It n is not a charactristic of R then

there exists . m > 0 such that m.1 = 0 But n . 1 = 0 ⟹ m = n

.Thus n is the charactristic of R

:Theorem :2-13

Let (F , + , .) be a field . Then the characteristic of .F it eitler 0 or a prime number P

:Proof

Let 0 ≠ n ∈ F be a characteristic of F, then n . 1 = 0 .

. Let n = $n1. n2$, $n1$ < n , $n2$< n . Then (1. $n2$)

yields ($n1$. 1) ($n2$. 1) = 0 16 But since F , $0 = 1$
. is a field

Thus either $n1$. 1 = 0 or $n2$. 1 = 0

Thus either $n1$. 1a = 0 $\Longrightarrow$ $n1$. a = 0 or $n2$. 1a = 0

$n2$ .a = 0 $\forall$ a $\in$ R thus the characteristic of F is $\Longrightarrow$
. ≤ max (1 , $n2$) < contradiction

.Thus n is a Prime