

(Recall that $\sin(2x) = 2 \sin x \cos x$.)

$$= -2 \sin(2x) + \sin(2x)$$

$$= -\sin(2x)$$

8- Derivatives of exponential and logarithmic functions

Exponential functions and their corresponding inverse functions, called logarithmic functions, have the following differentiation formulas:

(1) If $f(x) = e^x$, then $f'(x) = e^x$.

(2) If $f(x) = a^x$, $a > 0$, $a \neq 1$, then $f'(x) = (\ln a) \cdot a^x$.

(3) If $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$.

(4) If $f(x) = \log_a x$, $a > 0$, $a \neq 1$, then $f'(x) = \frac{1}{(\ln a) \cdot x}$.

Example : Find $f'(x)$ if $f(x) = e^{x^2 + 5}$

$$f'(x) = e^{x^2 + 5} \cdot 2x$$

$$f'(x) = 2x \cdot e^{x^2 + 5}$$

Example :

Find y' if $y = 5^{\sqrt{x}}$.

$$\begin{aligned}y' &= (\ln 5) \cdot 5^{\sqrt{x}} \cdot \frac{1}{2} x^{-1/2} \\ &= (\ln 5) \cdot 5^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \\ y' &= \frac{(\ln 5) \cdot 5^{\sqrt{x}}}{2\sqrt{x}}\end{aligned}$$

Example :

Find $f'(x)$ if $f(x) = \ln(\sin x)$.

$$\begin{aligned}f'(x) &= \frac{1}{\sin x} \cdot \cos x \\ &= \frac{\cos x}{\sin x} \\ \underline{f'(x) = \cot x}\end{aligned}$$

Example :

Find $\frac{dy}{dx}$ if $y = \log_{10}(4x^2 - 3x - 5)$.

$$\frac{dy}{dx} = \frac{1}{(\ln 10)(4x^2 - 3x - 5)} \cdot (8x - 3)$$

$$\frac{dy}{dx} = \frac{8x - 3}{(\ln 10)(4x^2 - 3x - 5)}$$

Chapter Five

1- Integration Rules

Here are the most useful rules:

Common Functions	Function	Integral
Constant	$\int a \, dx$	$ax + C$
Variable	$\int x \, dx$	$x^2/2 + C$
Square	$\int x^2 \, dx$	$x^3/3 + C$
Reciprocal	$\int (1/x) \, dx$	$\ln x + C$
Exponential	$\int e^x \, dx$	$e^x + C$
	$\int a^x \, dx$	$a^x/\ln(a) + C$
	$\int \ln(x) \, dx$	$x \ln(x) - x + C$
Trigonometry (x in <u>radians</u>)	$\int \cos(x) \, dx$	$\sin(x) + C$
	$\int \sin(x) \, dx$	$-\cos(x) + C$
	$\int \sec^2(x) \, dx$	$\tan(x) + C$

Rules	Function	Integral
Multiplication by constant	$\int cf(x) \, dx$	$c \int f(x) \, dx$
Power Rule ($n \neq -1$)	$\int x^n \, dx$	$x^{n+1}/n+1 + C$
Sum Rule	$\int (f + g) \, dx$	$\int f \, dx + \int g \, dx$
Difference Rule	$\int (f - g) \, dx$	$\int f \, dx - \int g \, dx$

Example:

What is $\int 6x^2 dx$?

We can move the 6 outside the integral:

$$\int 6x^2 dx = 6\int x^2 dx$$

And now use the Power Rule on x^2 :

$$= 6x^3 / 3 + C$$

Simplify:

$$= 2x^3 + C$$

Example :

What is $\int (8z + 4z^3 - 6z^2) dz$?

Use the Sum and Difference Rule:

$$\int (8z + 4z^3 - 6z^2) dz = \int 8z dz + \int 4z^3 dz - \int 6z^2 dz$$

Constant Multiplication:

$$= 8\int z dz + 4\int z^3 dz - 6\int z^2 dz$$

Power Rule:

$$= 8z^2/2 + 4z^4/4 - 6z^3/3 + C$$

Simplify:

$$= 4z^2 + z^4 - 2z^3 + C$$