$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$

$(\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\,si$

$\cos 3\theta + i\sin 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta + i(3\cos^2\theta - \sin^3\theta)$

$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

قانون الحد العام

$$(x+y)^n = \frac{n!}{(r-1)!\,(n-(r-1))!}x^{n-(r-1)}y^{r-1}$$

$x^{n-(r-1)}y^0 + x^{n-1}y^1 + \cdots + x^1y^{n-1} + x^0y^n \quad , \quad r = 1,2,\ldots,n$

$$\frac{n!}{r!\,(n-r)!}x^{n-r}y^r \quad , \quad r = 0,1,2,\ldots,n$$

$= x^ny^0 + x^{n-1}y^1$

EX:

$$(x+y)^7 = \frac{n!}{(r-1)!\,(n-r+1)!}x^{n-r+1}y^{r-1}$$

$$= \frac{7!}{(1-1)!\,(7-1+1)!}x^{7-1+1}y^{1-1} + \frac{7!}{(2-1)!\,(7-2+1)!}x^{7-2+1}y^{2-1} + \frac{7!}{2!\,5!}xy + \cdots$$

$$= \frac{7!}{7!}x^7y^0 + \frac{7!}{6!}x^6y + \frac{7!}{2!\,3!}x^5y^2 + \frac{7!}{3!\,4!}x^4y^3 + \frac{7!}{4!\,3!}x^3y^4 + \frac{7!}{5!\,2!}x^2y^5 + \frac{7!}{6!}xy^6 + \frac{7!}{7!}x^0y^7$$

$$(x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

يستخدم هذا القانون لايجاد قيم $\sin n\theta$ و $\cos n\theta$ او كلاهما باستخدام صيغة ديموفري وكما في المثال التالي:

EX:

جد قيمة $\sin 4\theta$ و $\cos 4\theta$ بدلالة $\sin\theta$ و $\cos\theta$

$(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$

$= \cos^4\theta + 4i\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta + \sin^4\theta$

$\therefore \cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$

$$r = |z| = \sqrt{x^2 + y^2} = \sqrt{4+9} = \sqrt{13}$$

$$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{13}} \quad , \quad \sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$$

EX:

$$z = \frac{1 + i\sqrt{3}}{2}$$

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$$

القوى والجذور

مبرهنة دي مورفي

$$z^n = r^n(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + i\sin n\theta)$$

Proof:

$$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$$

$$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$$

.

.

.

$$z_n = r_n(\cos\theta_n + i\sin\theta_n)$$

Let $z_1 = z_2 = \cdots z_n$

$$z_n = r^n(\cos\theta + i\sin\theta)^n$$

$$z_n = r^n(\cos n\theta + i\sin n\theta)$$

$$= (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$$

$$= \cos^2\theta + i\cos\theta\sin\theta + i\cos\theta\sin\theta + i^2\sin^2\theta$$

$$= (\cos\theta + i\sin\theta)^2$$

$$= \cos 2\theta + i\sin 2\theta$$

EX:

ايجاد $\cos 3\theta$ بدلالة [illegible] $\sin\theta , \cos\theta$ حسب طريقة ديمورفي

الصيغة القطبية للعدد المعقد

الاحداثيات (x, y) تقابلها بالصيغة القطبية (r, θ) حيث

$$z = x + iy \quad , \quad z = r(\cos\theta + i\sin\theta)$$

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

يمثل r طول المتجه أي ان $|z| = r$

θ الزاوية المحصورة بين المتجه الاحداثي السيني حيث $\theta = \arg(z)$

صيغة اويلر

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$-\pi < \arg z < \pi$$

اهم خاصية تتمتع فيها الزوايا هي :

1. $arg(z_1 z_2) = arg z_1 + arg z_2$

Proof:

let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 . z_2 = r_1 r_2[\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i(\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1)]$

$= r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

$\therefore \arg(z_1 z_2) = \theta_1 + \theta_2 = arg z_1 + arg z_2$

2. $\arg\left(\frac{z_1}{z_2}\right) = arg z_1 - arg z_2$

Proof:

$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}\left(\frac{\cos\theta_2 + i\sin\theta_2}{\cos\theta_2 + i\sin\theta_2}\right)$$

$$= \frac{r_1}{r_2}.\frac{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + i(-\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1)}{\cos^2\theta_2 + \sin^2\theta_2}$$

$$= \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i(\sin\theta_2\cos\theta_1 - \sin\theta_2\cos\theta_2)]$$

$$= \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

$$\therefore arg\frac{z_1}{z_2} = \theta_1 - \theta_2 = arg z_1 - arg z_2$$

EX:

$z = 2 + 3i$