$=|z_1|^2 + 2Re(u) + |z_2|^2 = |z_1|^2 + 2Re(z_1 . \bar{z}_2) + |z_2|^2$

$\leq |z_1|^2 + 2|z_1\bar{z}_2| + |z_2|^2$

$=|z_1|^2 + 2|z_2||\bar{z}_2| + |z_2|^2$

$=(|z_1| + |z_2|)^2$

$\therefore |z_1 + z_2| \leq |z_1| + |z_2|$

ويمكن تعميم هذه المبرهنة الى اكثر من عددين معقدين

$|z_1 + z_2 + z_3| \leq |z_1| + |z_2| + |z_3| + \cdots$

9- $|z_1 - z_2| \geq |z_1| - |z_2|$

$|z_1 - z_2|^2 = (z_1 - z_2)(\overline{z_1 - z_2}) = (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$

$=z_1\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_1 + z_2\bar{z}_2$

Let $u=z_1\bar{z}_2 \Rightarrow \bar{u} = \overline{z_1\bar{z}_2} = \bar{z}_1 . \bar{\bar{z}}_2 = \bar{z}_1 . z_2$

$\therefore |z_1 - z_2|^2 = |z_1|^2 - u - \bar{u} + |z_2|^2$

$=|z_1|^2 - 2Re(u) + |z_2|^2 \geq |z_1|^2 - 2|z_1\bar{z}_2| + |z_2|^2$

$=|z_1|^2 - 2|z_1||\bar{z}_2| + |z_2|^2$

$=|z_1|^2 - 2|z_1||z_2| + |z_2|^2$

$=(|z_1|^2 - |z_2|^2)^2$

$\therefore |z_1 - z_2| \geq |z_1| - |z_2|$

2- $|z| = |\bar{z}|$

$|z|^2 = z.\bar{z} = \bar{\bar{z}}.\bar{z} = \bar{z}.\bar{\bar{z}} = |\bar{z}|^2$

3- $|z_1 - z_2| = |z_2 - z_1|$

$|z_1 - z_2| = |-(z_2 - z_1)| = |z_2 - z_1|$

4- $\frac{1}{2}(z + \bar{z}) = \frac{1}{2}(x + iy + x - iy)$

$= \frac{1}{2}(2x) = x$

$= \sqrt{x^2} \leq \sqrt{x^2 + y^2} = |z|$

5- $\frac{1}{2i}(z.\bar{z}) = \frac{1}{2i}(x + iy - (x - iy))$

$= \frac{1}{2i}(2iy) = y$

$= \sqrt{y^2} \leq \sqrt{x^2 + y^2} = |z|$

6- $|z_1.z_2| = |z_1||z_2|$

$|z_1 z_2|^2 = (z_1 z_2.\overline{z_1 z_2}) = z_1 z_2.\bar{z}_1.\bar{z}_2$

$= \sqrt{z_1.z_2.\bar{z}_1.\bar{z}_2}$

$= \sqrt{z_1.\bar{z}_1.z_2.\bar{z}_2}$

$= \sqrt{z_1.\bar{z}_1}.\sqrt{z_2.\bar{z}_2}$

$= |z_1|.|z_2|$

$\therefore |z_1.z_2| = |z_1|.|z_2|$

7- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \qquad \ni |z_2| \neq 0$

$\left|\frac{z_1}{z_2}\right|^2 = \frac{z_1}{z_2}\left(\overline{\frac{z_1}{z_2}}\right) = \frac{z_1}{z_2}.\frac{\bar{z}_1}{\bar{z}_2} = \frac{|z_1|^2}{|z_2|^2} = \left(\frac{|z_1|}{|z_2|}\right)^2$

$\therefore \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

8- $|z_1 + z_2| \leq |z_1| + |z_2| \quad \forall z_1, z_2 \in \mathbb{C}$ المتباينة المثلثية

$|z_1 + z_2|^2 = (z_1 + z_2)(\overline{z_1 + z_2})$

$= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$

$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2$

نفرض أن $u = z_1.\bar{z}_2$ فإن

$\bar{u} = \overline{z_1\bar{z}_2} = \bar{z}_1\bar{\bar{z}}_2 = \bar{z}_1.z$

$\therefore |z_1 + z_2|^2 = |z|^2 + u + \bar{u} + |z_2|^2$

تعريف القيمة المطلقة للعدد المعقد z=x+iy بأنها $|z| = \sqrt{x^2 + y^2}$

$$|z| = \sqrt{z.\bar{z}} = \sqrt{x^2 + y^2}$$

Ex: $|z| = 1$

$\sqrt{x^2 + y^2} = 1 \Rightarrow \sqrt{(x-0)^2 + (y-0)^2} = 1$

$\sqrt{(x-x_0)^2 + (y-y_0)^2} = r$

Ex: $|z + 3| = 7$

$|x + iy + 3|$

$|(x + 3) + iy|$

$= \sqrt{(x+3)^2 + y^2} = 7$

المركز (-3,0)

r=7

Ex: $|z - 4 + 2i| = 5$

$|x + iy - 4 + 2i| = 5$

$|(x - 4) + i(y + 2)| = 5$

$\sqrt{(x-4)^2 + (y+2)^2} = 5$

المركز (4,-2)

خواص القيمة المطلقة

1- $|z| = \sqrt{z.\bar{z}}$

Proof

$|z|^2 = x^2 + y^2 = (x + iy)(x - iy) = z.\bar{z}$

$|z| = \sqrt{z.\bar{z}}$